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DAVID W. TAYLOR NAVAL SHIP RESEARCH AND DEVELOPMENT CENTER

Bethesda, Maryland 20084

SURFACE EFFECT SHIP AIR CAVITY
PRESSURE GAGES

by

Charles D. Bradley

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CENTRAL INSTRUMENTATION DEPARTMENT
DEPARTMENTAL REPORT

July 1981 DTNSRDC/CID-81/1

SURFACE EFFECT SHIP AIR CAVITY PRESSURE GAGES

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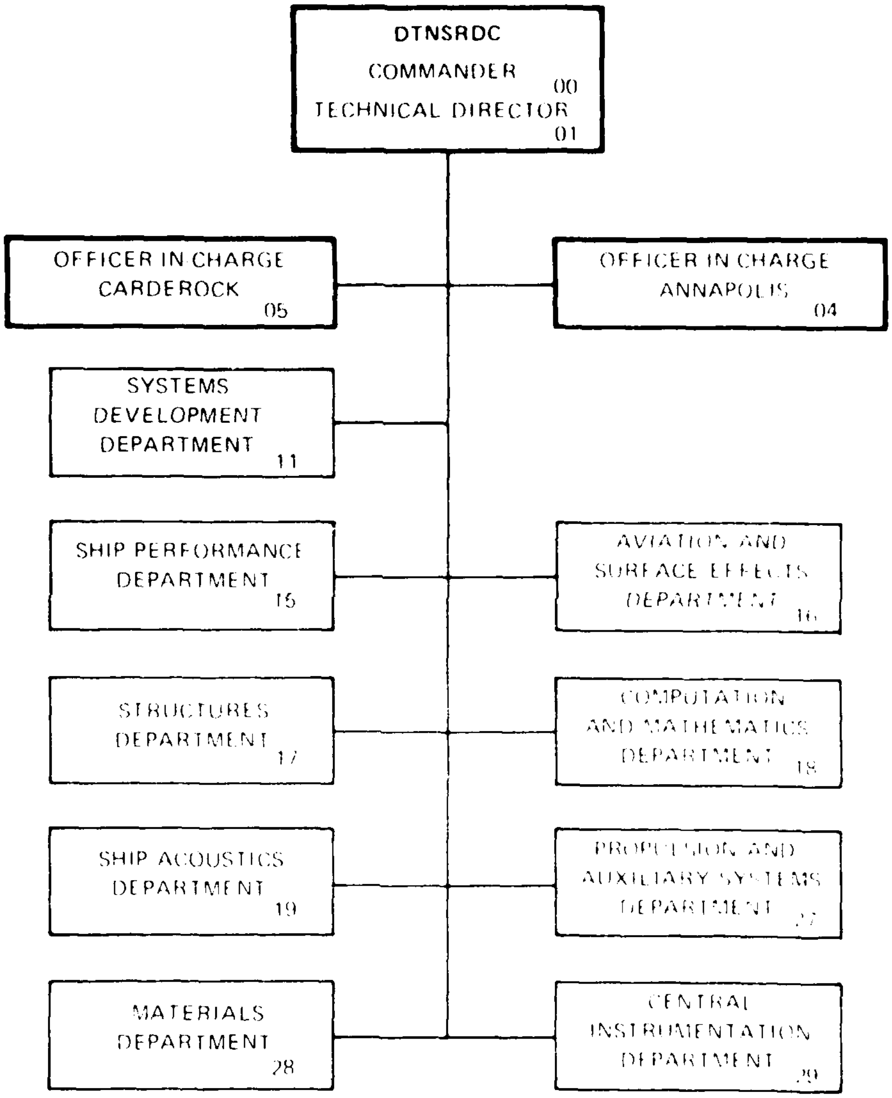

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20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

Attempts were made to extend the design of an existing waterproof, high frequency, foil strain gaged, flush diaphragm, 10 psi pressure gage to the 1 psi range. Three diaphragm configurations were investigated: uniform thickness, tapered thickness, and cantilever supported. Design equations for constructing these three types of diaphragms are given. From information derived from these equations, promising candidate pressure gages were built

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uniform thickness diaphragm gages made of various plastic materials, and a cantilever supported diaphragm gage with membrane coverings of two different thickness. Also one gage was built using a foil strain gage itself as the diaphragm. The results of the static calibrations and frequency response measurements are given for these gages. One experimental cantilever supported diaphragm gage looks promising for further development. Suggestions for improving its performance are given.

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ABSTRACT

Attempts were made to extend the design of an existing waterproof, high frequency, foil strain gaged, flush diaphragm, 10 psi pressure gage to the 1 psi range. Three diaphragm configurations were investigated: uniform thickness, tapered thickness, and cantilever supported. Design equations, for constructing these three types of diaphragms are given. From information derived from these equations, promising candidate pressure gages were built: uniform thickness diaphragm gages made of various plastic materials, and a cantilever supported diaphragm gage with membrane coverings of two different thickness. Also one gage was built using a foil strain gage itself as the diaphragm. The results of the static calibrations and frequency response measurements are given for these gages. One experimental cantilever supported diaphragm gage looks promising for further development. Suggestions for improving its performance are given.

ADMINISTRATIVE INFORMATION

This work was performed under the Center's IR/IED program, Program Element 62766N, Project Number F61412, Task Area Number ZF61412001.

INTRODUCTION

A 1 psi pressure transducer is needed to measure the pressure of water-saturated air in the cavities of Surface Effect Ships (SES) and SES models tested by the David Taylor Naval Ship R&D Center (DTNSRDC). In addition to a 1 psi full scale range the pressure gage should have the following characteristics: total non-linearity and hysteresis error less than one percent, a nominal sensitivity of 2×10^{-3} volts/volt excitation (this will be written as 2mv/v), waterproof, corrosion resistant, and high frequency response. The frequency response requirements for SES models fall into two distinct ranges: 0 to 20 Hz and 0 to 600 Hz. To minimize amplitude and phase distortion of the pressure signal, a pressure transducer should have its lowest resonant frequency at least five times higher than the highest frequency of interest. Therefore,

to measure 20 Hz, the gage should have no resonant frequency below 100 Hz; and to measure 600 Hz, no resonant frequency below 3000 Hz.

PRESENT GAGES, THEIR LIMITATIONS, AND PLANS TO IMPROVE ON THEM

CURRENTLY USED PRESSURE GAGES

At present two types of gages have been used to measure SES air cushion pressures.

Chamber Type Pressure Gage

The chamber type of transducer has a pressure-sensitive diaphragm inside of a chamber as shown in Figure 1. Opposite the diaphragm, the chamber has an opening that is connected by several feet of plastic tubing to the SES pressure cavity.

The chamber type pressure gage has several disadvantages:

Measurement Inaccuracy. If water accumulates in the tubing between the cavity and the pressure gage, the water causes a static pressure head which can create a pressure error of several percent for a 1 psi pressure measurement.

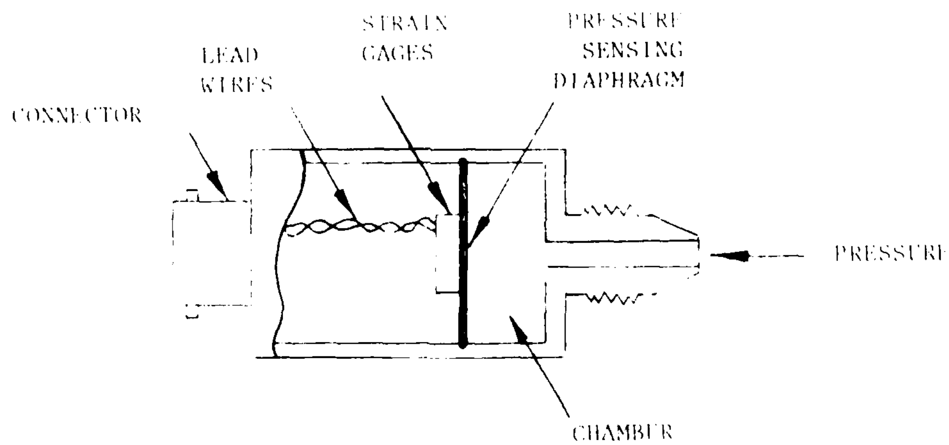

Figure 1 - Chamber Type Pressure Gage

Flow Distortion. A hole for the tubing must be drilled in the model's side panel to measure the pressure below the water line. The hole distorts the flow profile and hence the resulting pressure measurement.

Low Frequency Response. A chamber type transducer has an inherently low frequency response. As an example[1]* a chamber type pressure gage, when connected to the pressure environment with three feet of tubing, has a resonant frequency of 67 Hz. This is 33% below the minimum desired 100 Hz.

Waterproofing and Inconvenient Mounting. Because of their large size and mechanical construction, chamber type pressure transducers are not designed to be easily mounted in a SES model. In addition, these gage are not inherently waterproofed. Therefore, they must be located in dry areas or must be waterproofed during installation.

Corrosion. Gages made from non-corrosive materials must be selected and purchased. Since most chamber type pressure gages are not manufactured from corrosion resistant materials, special orders and/or higher costs result.

Flush Diaphragm Gage

Flush diaphragm gages have also been used for measuring SES air cushion pressures. These gages have the following desirable features.

Measurement Inaccuracy. The flush diaphragm allows the gage to be exposed directly to the water or saturated air in the SES cavity. Thus, there are no errors due to water clogged tubing.

No Flow Distortion. Since the diaphragm can be mounted flush with the SES side panel, the flow over the pressure gage installation remains undistorted.

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*A complete listing of references is given on page 61.

Frequency Response. The frequency response of a flush mounted diaphragm is limited only by the natural frequency of the diaphragm.

Corrosion. The flush diaphragm gage can be constructed of corrosion resistant materials.

Waterproofing and Mounting. Standard commerical 1 psi gages are not available as waterproofed units. And standard waterproofing techniques are not easily adapted to their configurations. In addition, these gages are difficult and expensive to mount since special adaptors and "O" ring seals are required.

Fragile. Commercial 1 psi gages are very fragile and typically during any one investigative program several are lost due to mishandling.

PURPOSE OF THESE INVESTIGATIONS

Several years ago the Central Instrumentation Department (CID) at DTNSRDC developed a pressure transducer known as the paddle gage or Bikini gage[2,3]. It can be manufactured for full scale pressures ranging between 10 and 1000 psi. The Bikini gage is completely waterproofed. Within the dimensional limitations of commercially available foil strain gages used for pressure measurements, the case of a Bikini type gage can be made in any one of several styles for easy mounting on SES models. The Bikini gage is rugged. It is constructed from beryllium copper (BeCu) a highly corrosion resistant material.

The purpose of these investigations was to extend the Bikini gage design to a full scale pressure range of 1 psi. Three types of diaphragms were investigated: the uniform thickness, the tapered thickness, and the cantilever supported. Equations for the theoretical pressure response for each type of diaphragm were developed. These equations allow examination of the diaphragm diameter, thickness, and resonant frequency. Based on these equations several promising candidate pressure gages were built--including four uniform thickness diaphragm gages made of various plastic materials and one cantilever supported diaphragm gage with membrane coverings of two different thickness. Also, an experimental pressure gage was built using a foil strain gage itself as the diaphragm. The section of this report titled UNIFORM THICKNESS DIAPHRAGM

contains (1) the design equations for the uniform thickness diaphragm gage, (2) considerations on selecting materials to be used in the gage constr... (3) a description of the uniform thickness experimental gages actually ... (4) the results of static and dynamic calibrations performed on these ex... gages, (5) a description of the gage built using the foil strain gage ... the diaphragm, and (6) the results of the calibration performed on this ... The section titled TAPERED THICKNESS DIAPHRAGM contains the design ... a tapered thickness gage. The section titled CANTILEVER SUPPORTED ... contains (1) several designs for cantilever supported gages, (2) a ... of the experimental gage actually built, and (3) the results of the ... performed on this experimental gage. The section titled SUMMARY AND ... contains (1) a summary of the performance of the experimental gages a... conclusions about their suitability as 1 psi transducers. The secti... RECOMMENDATIONS contains recommendations for improving the performance ... these experimental gages.

UNIFORM THICKNESS DIAPHRAGM

Figure 2 shows a circular diaphragm of uniform thickness typically ... pressure gages. In the Bikini gage, the diaphragm is usually machined ... integral part of the pressure gage body[3]. The uniform thickness diaphr... easier to machine than tapered shapes.

Standard foil strain gages, designed especially for measuring pres... when bonded to diaphragms, can be purchased from strain gage manufacture... The diaphragm strain gage is a four active arm strain gage bridge mounte... a single insulating backing material (see Figure 2). The two outer str... gage elements are called "radial" gages because they are sensitive to r... strain. The two inner elements measure "tangential" strain which is str... along circles concentric to the diaphragm's center.

SENSITIVITY

When pressure is applied to the transducer's pressure-sensing surf... Figure 2) the diaphragm is deflected downward. The radial elements of ...

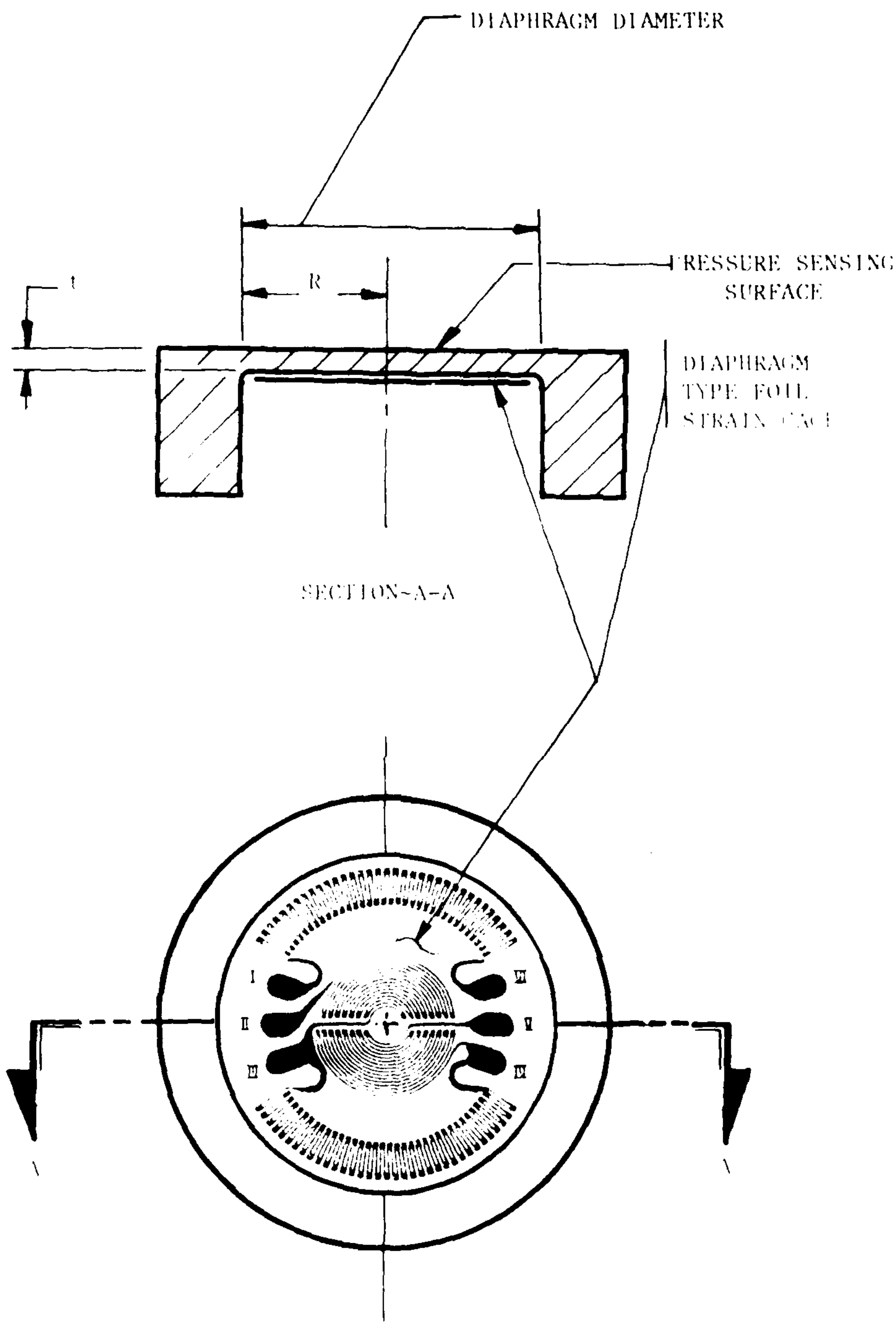


Figure 2 - Uniform Thickness Diaphragm Pressure Gage

diaphragm strain gage go into compression; the tangential elements go into tension. In Appendix A it is shown that the radial strain at the diaphragm's outer edge is:

$$e_r = -\frac{3PR^2(1-v^2)}{4t^2E} \tag{1}$$

where P = pressure in psi

R = diaphragm radius

t = diaphragm thickness

v = Poisson's ratio

E = modulus of elasticity

The tangential strain at the diaphragm's center is opposite in sign to e_r and is given by

$$e_t = +\frac{3PR^2(1-v^2)}{8t^2E} \tag{2}$$

These equations are based on the assumptions stated in Appendix A.

For any applied pressure, Equations (1) and (2) show that the magnitude of the radial strain is twice the magnitude of the tangential strain. This is an inherent characteristic of a diaphragm strain field.

Since the tangential and radial strains are different, it is desired to define an average strain, e_a. Average strain will be used to evaluate the sensitivity of various pressure gage designs. Appendix A shows that the average strain is

$$e_a = 0.4038\,\frac{PR^2(1-v^2)}{Et^2} \tag{3}$$

The average strain, e_a, does not appear uniformly in any element of the diaphragm gage. However, the output of the diaphragm gage can be estimated by assuming it behaves as a four active arm strain gage bridge with each gage in the bridge having a gage factor of 2 and each gage undergoing a uniform strain with a magnitude equal to $|e_a|$. With this assumption, if e_a equals about 1000 microinches/inch for the rated full scale pressure of the transducer, the transducer's output will be 2 mv/v, the sensitivity desired for the 1 psi pressure gage.

LINEARITY

Diaphragm strain is not in general a linear function of pressure. While Equations (1), (2), and (3) show strain is a linear function of pressure, these equations are only valid for small diaphragm deflections. The general rule[4] is: the deflection at the center of the diaphragm should not exceed the diaphragm thickness, t, if the non-linearity is to be limited to 1%. The equation for the center deflection, z, is[4]:

$$z = \frac{3PR^4(1-\nu^2)}{16Et^3} \qquad (4)$$

OPTIMIZING SENSITIVITY AND LINEARITY

Equations (3) and (4) may be combined to find the maximum average strain allowable which will maintain a maximum non-linearity error of 1%. First Equation (4) is divided by the thickness, t, to give the deflection-to-thickness ratio at the diaphragm center. To maintain the 1% non-linearity error, this ratio is set equal to 1.0:

$$1.0 = \frac{z_{max}}{t} = \frac{3}{16}\,\frac{P_{max}(1-\nu^2)}{E}\,\frac{R^4}{t^4} \qquad (5)$$

Equation (5) is solved for the ratio $\frac{R^4}{t^4}$ which is then substituted in Equation (3) to give:

$$(e_a^2)_{max} = \frac{16(0.4038)^2 \; P_{max}(1-v^2)}{3E} \tag{6}$$

Equation (6)* shows that, for a given full scale pressure of P_{max}, $(e_a)_{max}$ is the highest full scale strain that can be allowed while keeping the non-linearity error less than 1%. Equation (6) also shows strain levels (and hence transducer outputs) are highest using materials with a low modulus of elasticity. Referring back to Equation (3), note that the average strain could also be increased by making the diaphragm thinner. The following analysis will show that thinner diaphragms of materials having a higher modulus would produce a more non-linear output for the same average strain level. In Equation (3) suppose E_1 E but t_1 is adjusted so that $E_1 t_1^2 = Et^2$. Using this result in Equation (5) we have

$$\frac{z_{max_1}}{t_1} = \frac{3}{16} \; \frac{P_{max}(1-v^2)}{E_1} \; \frac{R^4}{t_1^4} = \frac{3}{16} \; \frac{P_{max}(1-v^2)R^4}{(E_1 t_1^2)t_1^2} \; \frac{(t^2)}{(t^2)}$$

$$= \frac{3}{16} \; \frac{P_{max}(1-v)^2}{Et^2} \; \frac{R^4}{t_1^2} \; \frac{(t^2)}{(t^2)} = \frac{z_{max}}{t} \; \frac{t^2}{t_1^2}$$

---

*In Equation (3), the output strain, e_a, is linearly proportional to pressure. Equation (6) appears to show that the strain is proportional to the square root of pressure. Actually Equation (6) only relates the maximum strain, $(e_a)_{max}$, to a desired maximum pressure, P_{max}, to give a non-linearity error less than 1%.

Since $t > t_1$, $z_{max_1}/t_1 > z_{max}/t = 1$. Since $z_{max}/t = 1$ was chosen as the largest deflection to thickness ratio which would keep the non-linearity error less than 1%, the deflection ratio $z_{max_1}/t_1 > 1$ will result in a non-linearity error greated than 1%. Also, to make a 1 psi transducer it is much more difficult to machine extremely thin diaphragms of .002 to .003 inch associated with high modulus materials.

FREQUENCY RESPONSE

Subject to the assumptions listed in Appendix A, the undamped resonant frequency of a rigidly clamped diaphragm in air can be expressed as follows:

$$f_n = 0.471 \frac{t}{R^2} \left(\frac{gE}{\gamma(1-\nu^2)} \right)^{1/2} \text{ Hz} \qquad (7)$$

where g = acceleration of gravity (386.4 in/sec^2)

γ = specific weight of the diaphragm material in lbs/in^3

Equation (7) shows that to get a high resonant frequency, it is desirable to have a thick diaphragm of minimum radius made of materials having a low specific weight (that is, a low density) and a high elastic modulus. The resonant frequency requirements for a high modulus and thicker diaphragm run counter to the sensitivity requirements for a low modulus and thin diaphragm. A compromise must be made between a high sensitivity and high natural frequency. However, once a sensitivity and natural frequency are selected, the linearity can be improved by selecting a diaphragm material with a low elastic modulus and simultaneously increasing the diaphragm thickness to keep the product Et^2 constant. This can be seen by first noting that Equation (3) shows $e_a \propto 1/Et^2$ (where the symbol $\propto$ means "is proportional to"), while Equation (7) shows $f_n^2 \propto Et^2$. Thus, the sensitivity and natural frequency remain the same if the product Et^2 remains fixed. Next, an analysis similar to the one at the top of this page can be performed to show that reducing the ratio of z_{max}/t to less than one will reduce the non-linearity error.

POTENTIAL DIAPHRAGM MATERIALS

Metals

To obtain the highest sensitivity, linearity, and natural frequency, the characteristics of several metals were compared in Table 1. Of the materials compared, magnesium is the metal with both the lowest modulus and density. However, magnesium has very poor corrosion resistance in water[5]. Of the remaining materials beryllium copper (BeCu) has the best corrosion resistance in water. It is used at DTNSRDC for making waterproof Bikini pressure gages with full scale pressure ranges of 10 psi and above.

TABLE 1 - IMPORTANT PROPERTIES OF METALS CONSIDERED FOR PRESSURE GAGE DIAPHRAGMS

Material	Elastic Modulus, psi	Specific Weight lbs/in^3	Corrosion Resistance of Water
Beryllium Copper	19×10^6	.297	very good
Titanium	16×10^6	.164	good
Aluminum	10×10^6	.101	fair
Magnesium	6.5×10^6	.0648	poor

For a 1 psi gage with a diaphragm size of 1/2 inch diameter, Table 2 gives design data on the diaphragm thickness, maximum allowable average strain, and natural frequency for four candidate metals to be used in gage construction. The data in Table 2 shows that although the natural frequencies are acceptable (1) the maximum allowable strains are well below the desired 1000 microinches/inch to give a sensitivity of 2 mv/v and (2) the diaphragm thickness are too thin to make. The DTNSRDC Machine Shop states that they cannot reliably machine a 1/2 inch diaphragm thinner than 0.004 inch or a 1/4 inch diaphragm thinner than 0.002 inch.

TABLE 2 - DESIGN DATA: METAL DIAPHRAGMS FOR 1 PSI GAGES

Material	Diaphragm Thickness, in.	Maximum allowable strain, e_a, for 1% Non-linearity Microstrain	Natural Frequency, Hz
BeCu	.0024	220	3010
Ti	.0026	230	4020
Al	.0028	280	4450
Mg	.0032	350	5070

Due to these considerations, no uniform thickness diaphragms were machined from any of the metals listed in Table 1.

Plastics

Since the theoretical equations show that a low density, low modulus diaphragm material is desirable, two plastics were investigated as diaphragm materials. They were unlubricated delrin* (a homopolymer acetal) and unplasticized polyvinyl chloride (PVC), type 1. The acetal was picked because it was claimed[6] to have more metallic-like properties (high fatigue endurance limit, dimensional stability, resistance to bending under load, and returns to its original shape when a load is removed) than most plastics. Also, it has a low water absorption rate. However, it is hard to cement a strain gage to it (a cyanogen cement is required). The unplasticized PVC was chosen since it is very rigid, has a low water absorption rate and cements adhere to it easily. The modulii and specific weights of acetal and PVC are shown in Table 3.

TABLE 3- IMPORTANT PROPERTIES OF PLASTICS CONSIDERED FOR PRESSURE GAGE DIAPHRAGMS

Material	Modulus psi	Specific Weight lbs/in.
Acetal	0.41×10^6	.051
PVC	0.43×10^6	.051

*Trademark of Dupont

For a 1 psi gage, Table 4 shows design data for the thickness, average strain level, and natural frequency of 1/2 inch diameter acetal and PVC diaphragms. In developing Table 4, calculations using Equation (6) show that the non-linearity will be less than 1% for strain levels well over 1000 microstrain. Therefore, using Equation (3) the diaphragm thickness was calculated to produce 1000 microstrain at 1 psi.

TABLE 4 - DESIGN DATA: PLASTIC DIAPHRAGMS FOR 1 AND 5 PSI GAGES

Pressure Range	Material	Diaphragm Thickness in.	Average Strain e_a, for 1 psi Microstrain	Natural Frequency Hz
1 psi	Acetal	.0070	1000	3324
1 psi	PVC	.0068	1000	3324
5 psi	Acetal	.0156	1000	7432
5 psi	PVC	.0152	1000	7432

In addition, the table shows the same design data for 5 psi pressure transducers. Since DTNSRDC's dynamic pressure calibrator does not operate below 5 psi, it was decided to build some 5 psi gages in addition to the 1 psi gages to check the correlation between calculated and measured values of natural frequency.

STATIC CALIBRATION

Since the design calculations showed good potential for fabricating plastic pressure gages, four plastic units were built in the range of 1 to 5 psi. Two acetal pressure gages were made with a range of 5 psi. Although a number of attempts were made, no successful 1 psi pressure gages were manufactured from the available acetal stock. Two PVC pressure gages were made: one with a full scale range of 1 psi, and one with a range of 2 psi. The four pressure gages were statically calibrated. And a linear regression analysis was performed on each calibration run. The results are shown in Table 5. The sensitivity

of each gage was far below the design calculations of 2 mv/v. The combined non-linearity and hysteresis of every run varied from 1.30% to 4.33%. These values are well above the design limit of 1%. Also, the zero shift of each gage was high, approaching 6% of full scale in some cases. The reduced sensitivity probably was caused by the thickness and modulus of elasticity of the strain gage and adhesive. For thin diaphragms the strain gage and adhesive increase the diaphragm thickness significantly. Also the strain gage and adhesive have moduli of elasticity which can be greater than the moduli of PVC and/or acetal.

TABLE 5 - CALIBRATION RESULTS FOR FOUR PLASTIC PRESSURE GAGES

Pressure Range	Diaphragm Thickness in.	Calibration Run No.	Full Scale Output mv/v	Combined Non-Linearity and Hysteresis: Maximum Deviation % F.S.	Combined Non-Linearity and Hysteresis: Standard Deviation psi	Zero Shift % F.S.
5 psi Acetal	.0180	1	.7683	-1.52	.0505	1.04
		2	.7641	2.22	.0932	3.79
		3	.7448	1.91	.0711	3.62
		4	.7329	-2.41	.0948	3.68
		average=	.7525			
5 psi Acetal	.0162	1	.8371	-3.87	.1380	5.85
		2	.8314	-3.64	.1335	5.58
		3	.8295	-3.56	.1281	5.50
		average=	.8327			
2 psi PVC	.0105	1	.8108	1.30	.0123	4.46
		2	.7673	3.37	.0511	5.52
		average=	.7890			
1 psi PVC	.006	1	.5345	-4.33	.0307	5.76
		2	.5115	-3.20	.0220	2.93
		average=	.5230			

DYNAMIC CALIBRATION

The two 5 psi acetal gages were dynamically calibrated in a Type D-142 dynamic calibrator developed by the Navy Metrology Laboratory, Pomona, CA.

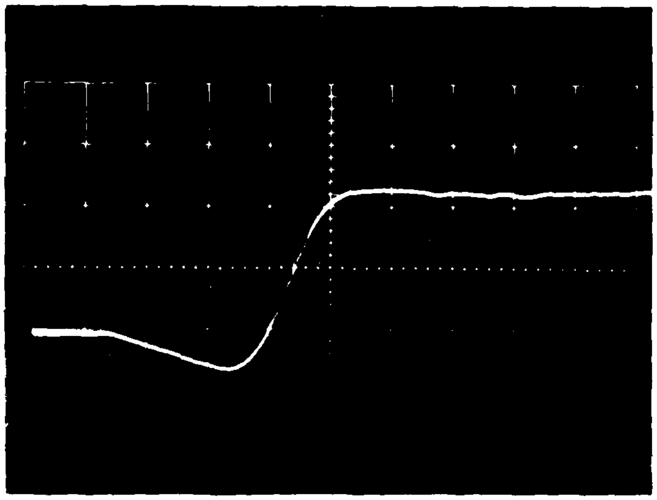
The calibrator generates a step pressure change on the face of a gage using helium gas. A piezoelectric pressure transducer, Kistler, Type 603A, is used as a reference device to measure the rise time of the pressure step. Figure 3 shows the pressure step generated by the calibrator; Figures 4 and 5 show the pressure response of the two acetal pressure transducers. The gages natural frequencies were derived from these pressure response photographs and are listed in Table 6. The calculated natural frequency from design equations are also listed in Table 6 for comparison. The data indicates the measured frequencies are 23 to 38 percent higher than the calculated values. This difference is probably due to the stiffening effects of the strain gage and adhesive on the diaphragm.

TABLE 6 - CALCULATED AND MEASURED NATURAL FREQUENCIES OF 5 PSI ACETAL PRESSURE GAGES

Pressure Gage	Diaphragm Thickness in.	Calculated f_n Hz	Measured f_n Hz	Percent by which measured f_n exceeds calculated f_n
Acetal	.0180	9063	12,500	38
Acetal	.0162	8157	10,000	23

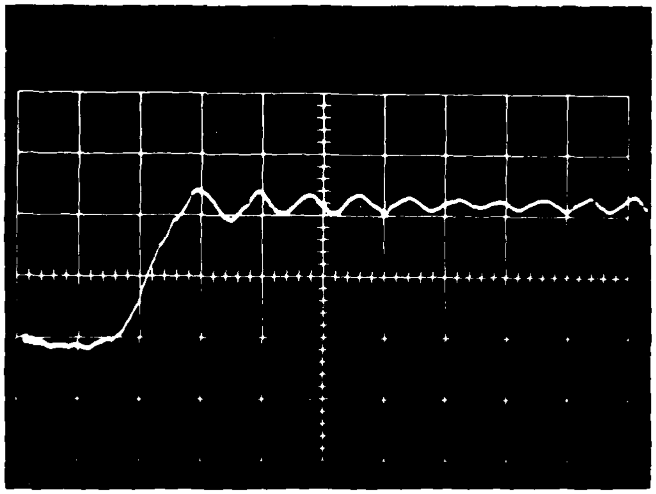
ZERO THICKNESS DIAPHRAGM PRESSURE GAGE

Since the outputs of the plastic pressure gages were so low, it was decided to examine the influence of the foil strain gage on pressure transducer sensitivity. A diaphragm type foil strain gage (1/2 in. nominal diameter) was cemented across the open end of a hollow cylinder as shown in Figure 6. Since there is no diaphragm except the strain gage itself, the unit is called the "zero thickness" diaphragm pressure gage. The unit was calibrated in the same manner as the other pressure gages over a range of 0 to 0.4 to 0 psi. The calibration was stopped at 0.4 psi to prevent over-stressing the "diaphragm". As Figure 7 shows, the gage was highly non-linear. However, the hysteresis was only 0.8%, and the zero shift was 0.6%. A second calibration run, not shown on the graph, showed the repeatability was 0.7%.



Time - 0.1 msec/division

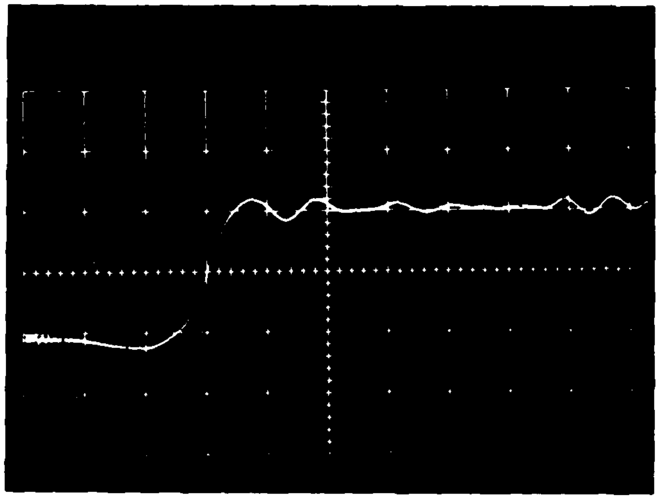
Figure 3 - Pressure Step Generated by the Dynamic Pressure Calibrator



Time - 0.1 msec/division

Figure 4 - Step Function of Pressure Gage made with 0.018 inch Acetal Diaphragm

2 PSI PRESSURE STEP



calculated
$f_n = 12.5KHz$

Time - 0.1 msec/division

NOTE: The increase in the oscillatory amplitude at the right of Figure 5 is an anomaly generated by the calibrator, not the pressure gage.

Figure 5 - Step Function Response of Pressure Gage made with 0.0162 inch Acetal Diaphragm

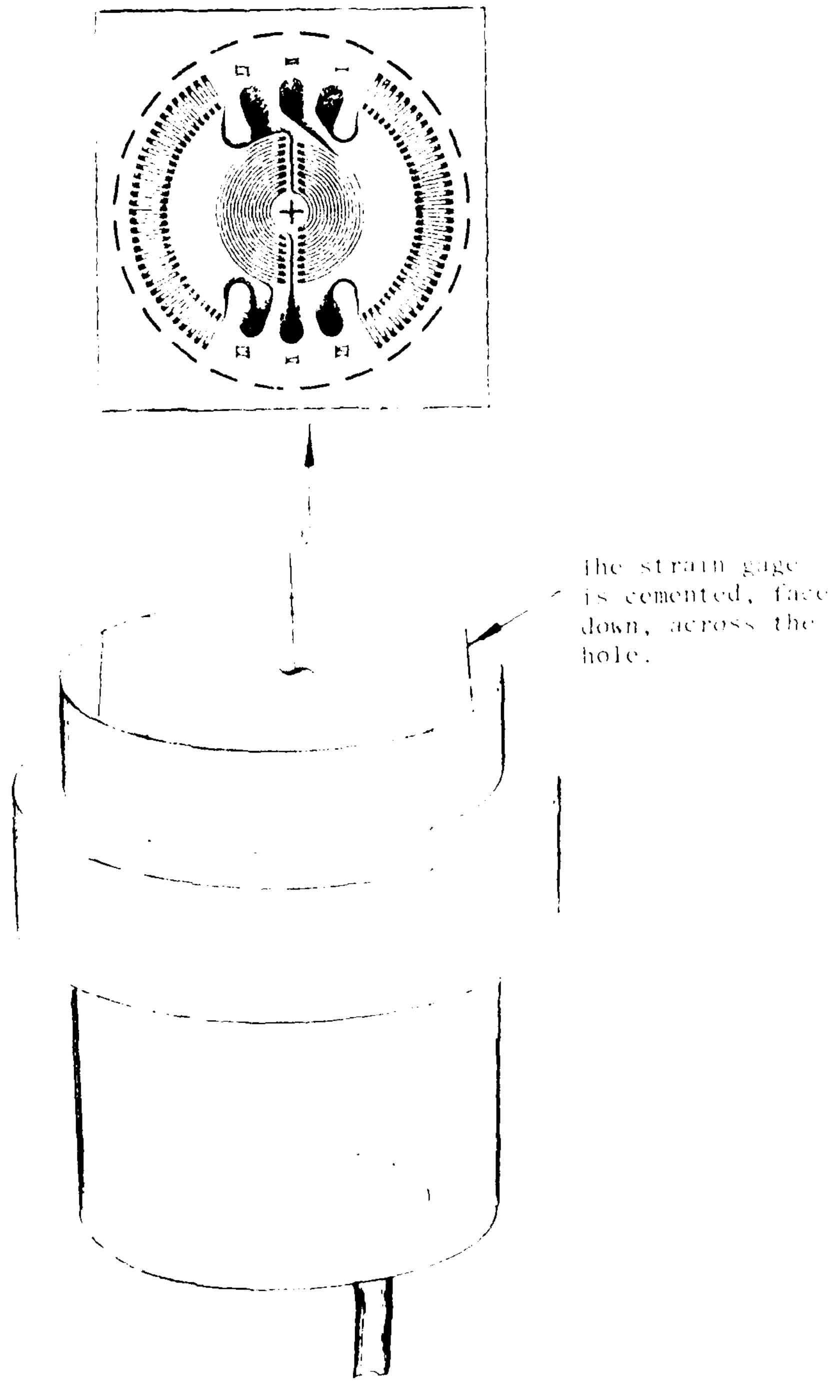


Figure 6. The "Zero Thickness" Diaphragm Pressure Gage

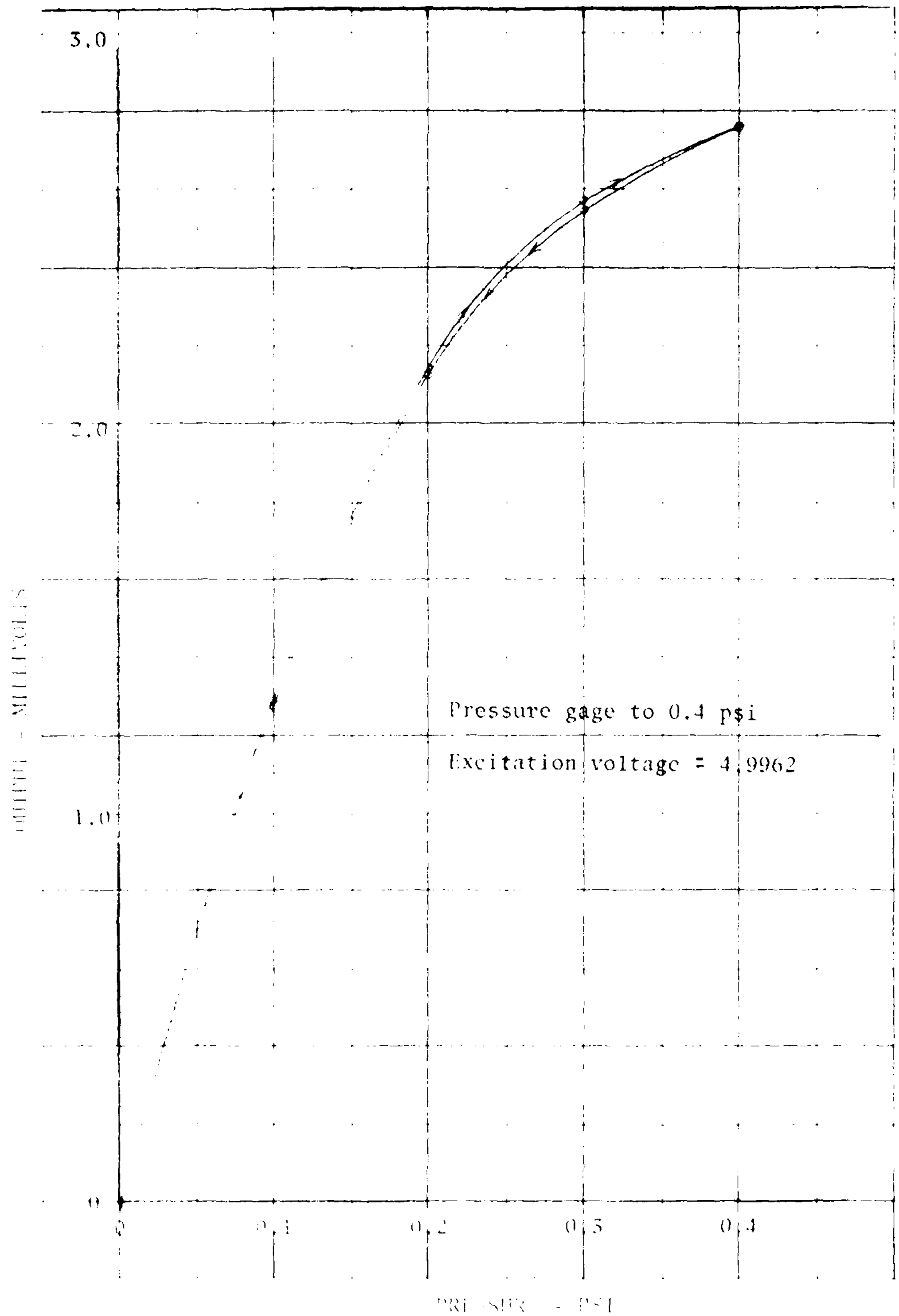


Figure 7 - Calibration of "Zero Thickness" Diaphragm to 0.4 PSI

The gage was also calibrated twice in its linear range from 0 to 0.1 psi, and the data was plotted as shown in Figure 8. A regression analysis was performed on the data and the results are tabulated in Table 7. The analysis showed that the sensitivity was 0.2453 mv/v for a full scale pressure of 0.1 psi. This is equivalent to a sensitivity of 2.453 mv/v psi (i.e. (.2453 mv/v)/(.1 psi) which is on the order of our design goal of 2 mv/v sensitivity for a 1 psi gage. However, the sensitivity was achieved with a "zero thickness" diaphragm. But, Equation (3) shows the sensitivity is inversely proportional to the square of the diaphragm thickness. Therefore, when the strain gage is actually bonded to diaphragm, the output of the pressure gage is reduced since the total thickness of diaphragm and strain gage is much thicker than the strain gage alone. Because of this decrease in sensitivity, we can conclude that a 1 psi full scale pressure transducer with a 1/2 inch diameter diaphragm cannot be made with a sensitivity of 2 mv/v using a diaphragm strain gage.

TABLE 7 - CALIBRATION RESULTS FOR "ZERO THICKNESS" DIAPHRAGM PRESSURE GAGE

Run No.	Sensitivity	Linearity and Hysteresis Maximum Deviation % of FS	Standard Deviation psi	Zero Shift % of FS
1	.2368	.84	.0004	.08
2	.2537	-1.37	.0007	.86
average sensitivity	.2453			

TAPERED THICKNESS DIAPHRAGM

A cross sectional view of a tapered thickness diaphragm is shown in Figure 9. Notice that the tapered diaphragm has a greater thickness at the diaphragm edge than at the center. All of the taper is on the inside surface of the diaphragm. The pressure sensitive surface is kept flat to prevent any distortion in the flow stream.

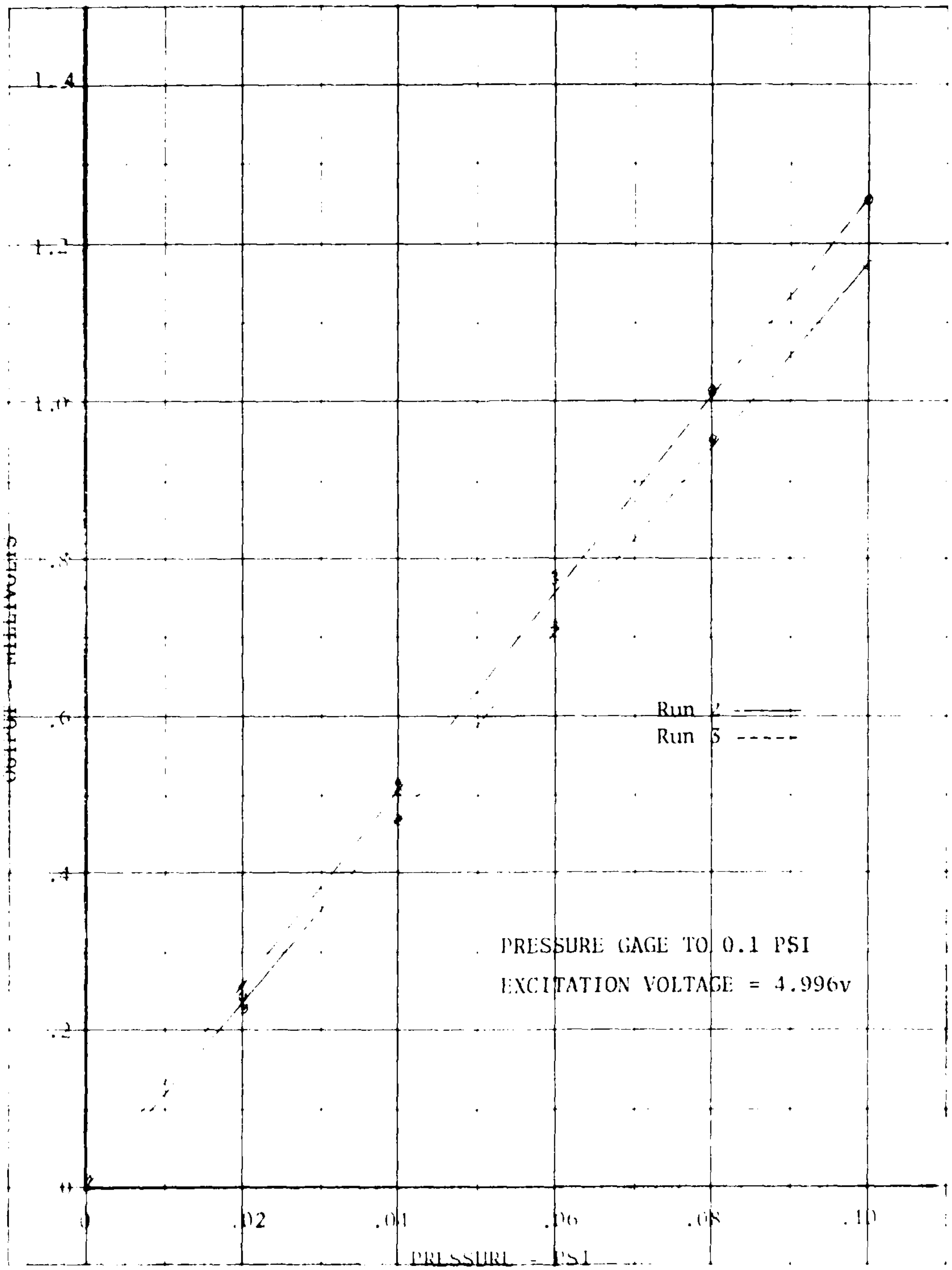


Figure 8 - Calibration of "Zero Thickness" Diaphragm to 0.1 PSI

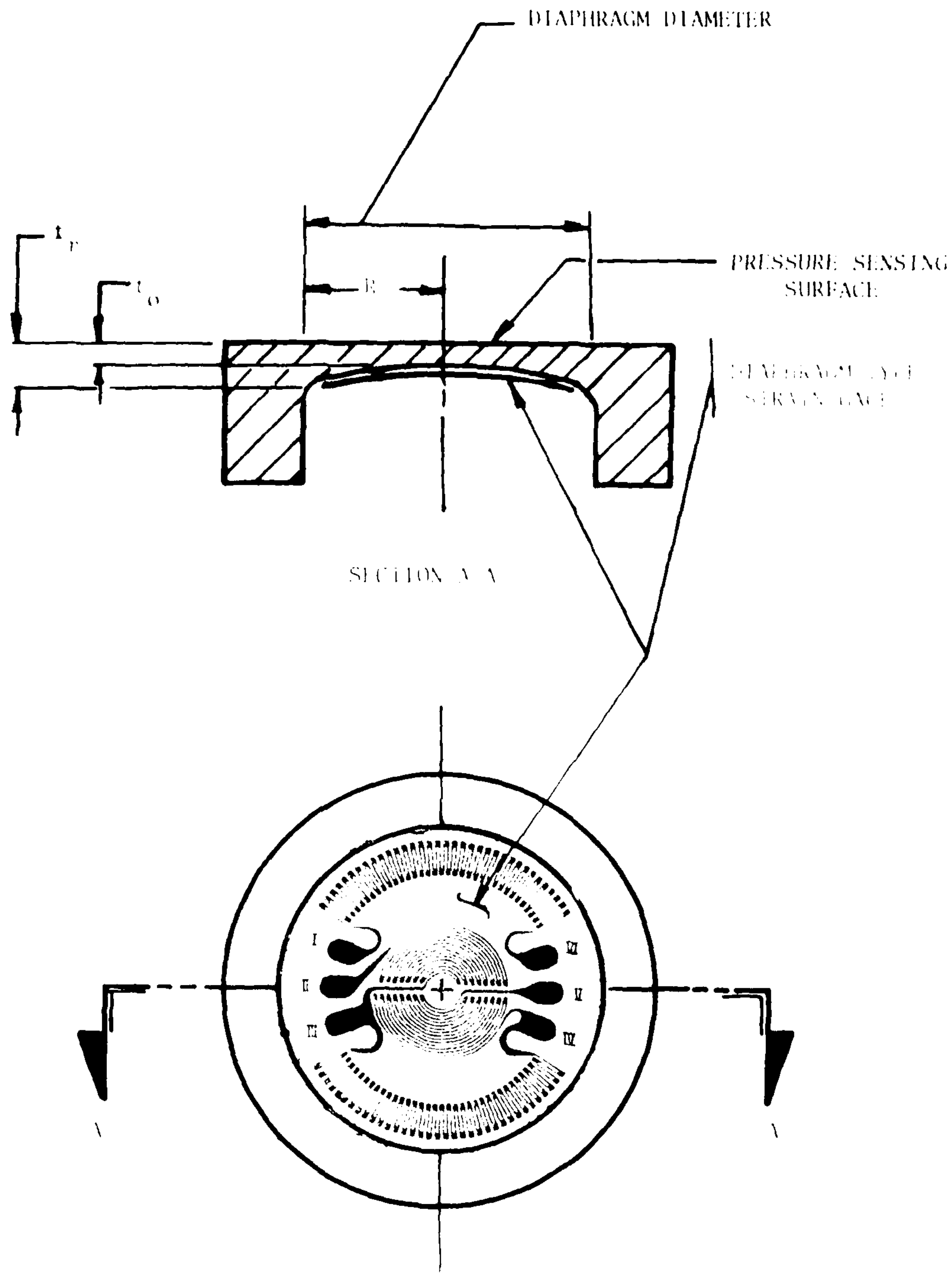


Figure 9 Tapered Thickness Pressure Transducer

The tapered thickness diaphragm was examined for two reasons. First, Equations (1) and (2) for uniform thickness diaphragms indicate that the radial strain is twice the tangential strain at any applied pressure. If the thickness at the diaphragm's edge, t_r, is greater than the center thickness, t_o, the magnitude of the radial strain would approach that of the tangential strain. Since the radial strain is so high in a uniform thickness diaphragm there is a danger of excessive strain in the diaphragm in the radial direction even for acceptable average diaphragm strains. The tapered thickness diaphragm, since it reduces the radial strain, reduces this possibility of excessive strain.

The second reason for examining the tapered thickness diaphragm is that the unit has relatively small mass at its center. The small mass combined with the thick, stiff circumference would indicate a higher natural frequency than for a uniform thickness diaphragm for the same pressure range.

The shape of the taper was chosen to conform to the mathematical expression:

$$t = t_o e^{-nx^2/6} \tag{8}$$

where t = thickness at x

x = r/R where r is the radial distance from the center of the diaphragm and R is the radius of the diaphragm

t_o = thickness at diaphragm center

n = constant that depends on the chosen taper (see Appendix B); n must be less than zero for pressure gage diaphragms.

The stress at the diaphragm center is:

$$s = bP\frac{R^2}{t_o^2} \tag{9}$$

and the deflection at the diaphragm's center is:

$$z = aP\frac{R^4}{Et_o^3} \tag{10}$$

where a and b are tabulated constants that depend on n and are found in Appendix B. These two equations can be combined as shown in Appendix B to yield:

$$e^2_{max} = \frac{b^2 P_{max}}{aE} \tag{11}$$

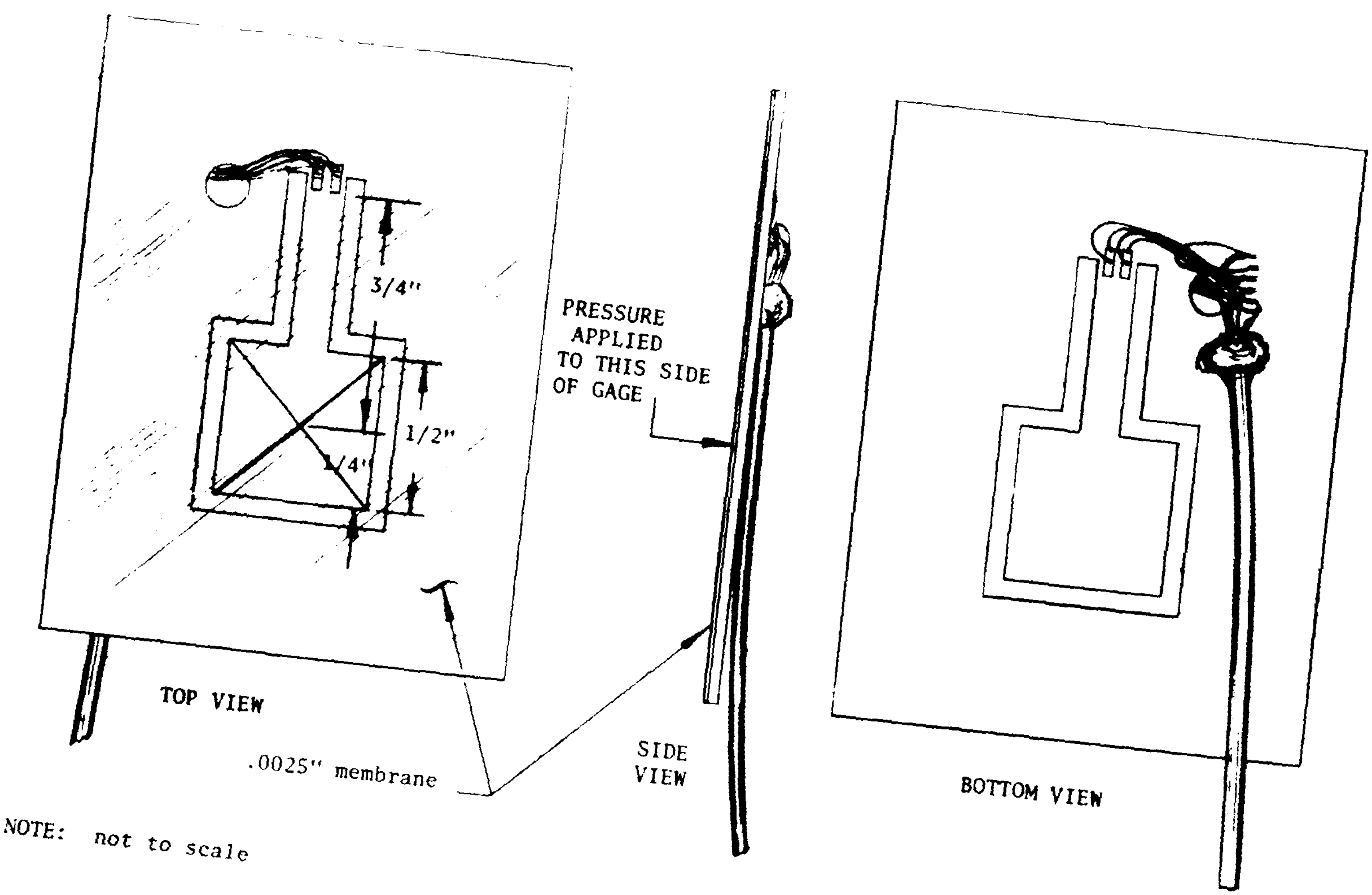
Equation (11) shows that, for a given full scale pressure (P_{max}), e_{max} is the highest diaphragm strain allowable which will keep the output linear within 1% of full scale. The equation shows that strains are highest using materials with a low modulus of elasticity, the same conclusion made for a uniform thickness diaphragm.

The table in Appendix B also shows, that for e_{max} = 500 microinches/inch (50% of our desired sensitivity) and n less than zero, the elastic modulus must be below 10 x 10^6 psi. Magnesium is the only metal which has a modulus this low. As was pointed out before, magnesium corrodes easily in water. Also, a magnesium diaphragm would be too thin. The center of the tapered diaphragm, t_0, would be less than .0032 inches (the thickness of a uniform thickness diaphragm gage made from magnesium - which we noted was too thin to reliably machine). If acetal and PVC were machined into tapered diaphragm pressure gages, the gages would exhibit the same drawbacks as uniform thickness diaphragm plastic gages: large hysteresis and zero shifts and low strain output. For these reasons, and because a tapered thickness diaphragm is difficult to machine, no tapered diaphragm gages were made.

CANTILEVER SUPPORTED DIAPHRAGM

THEORY

The cantilever supported diaphragm is quite different from the uniform and tapered thickness diaphragms. As seen in Figure 10, the cantilever supported diaphragm pressure gage is made from a single piece of thin metal stock. A slot in the shape of a "spoon" is cut in it. The wide portion of the spoon is a 1/2 inch X 1/2 inch diaphragm, and the slender section is the supporting



NOTE: not to scale

Figure 10 - Cantilever Supported Diaphragm Pressure Gage

cantilever spring. Strain gages are mounted on both sides of the cantilever 3/4 inch from the center of the diaphragm. The metal casing outside of the "spoon" may be any shape and thickness desired. The particular shape shown in Figure 10 was chosen to fit existing calibration equipment. After the strain gages were mounted the entire pressure gage was covered with a thin rubber membrane glued to the metal on the side opposite the connecting cable. There are other shapes of diaphragms and cantilevers that can be made as shown in Appendix C.

Pressure is applied to the membrane side of the unit. The diaphragm deflects, bending the cantilever and generating an electrical output from the strain gages. For the gage in Figure 10, it is shown in Appendix C that the strain at the strain gage location is given by:

$$\epsilon = p \frac{7.05}{Et^2} \tag{12}$$

For beryllium copper (BeCu) and a design strain of 1000 micro-inches/inch at 1 psi, the metal thickness, t, is 0.0221 inch. This is almost an order of magnitude thicker than a uniform thickness BeCu 1 psi diaphragm.

The cantilever supported diaphragm gage has a linear output, even at large strains, subject to the assumption in Appendix C. These assumptions are much less restrictive than the requirements to produce a non-linearity error less than 1% for uniform and tapered pressure gages.

To calculate the natural frequency of a cantilever pressure gage, the mass of both the diaphragm and the cantilever must be taken into account. The resonant frequency equation developed in Appendix C is:

$$f_n = 0.28 \left[\frac{EI}{(m + \frac{33}{140} \mu L) L^3} \right]^{1/2} \tag{13}$$

where I = moment of inertia of the cantilever

m = mass of the diaphragm

μ = mass per unit length of the cantilever, and

L = length of the cantilever

these terms are further defined in Appendix C.

For a BeCu pressure gage with the dimensions previously given, the natural frequency is 340 Hz. Obviously, this gage can not be used for measurements in the frequency range of 0 to 600 Hz. But it can be used for SES model measurements in the range of 0 to 20 Hz.

EXPERIMENTAL PRESSURE GAGE

One experimental cantilever pressure gage, with a design based on Equation (12) was constructed from beryllium copper. The unit was evaluated with: (1) no rubber covering, (2) a 1/16 (.0625) inch rubber membrane cover, and (3) a .0025 inch rubber membrane cover.

To calibrate the gage with no covering, weights were suspended from a string cemented to the center of the diaphragm. The weights were converted to equivalent psi for data reduction purposes. A regression analysis was performed on the data, and the results are shown in Table 8.

TABLE 8 - CALIBRATION RESULTS FOR CANTILEVER PRESSURE GAGE

Condition	Run No.	Sensitivity mv/v	Maximum Deviation % of FS	Standard Deviation PSI	Zero Shift % of Full Scale
no cover	1	2.5678	-.11	.0005	.01
.0625" rubber	2	1.1398	2.53	.0123	2.32
.0625" rubber	3	1.1247	-3.42	.0146	1.89
.0625" rubber	4	1.1559	-3.50	.0151	1.00
average sensitivity		1.1401	(average of runs 2, 3, 4)		
.0025" rubber	5	2.4379	-2.17	.0095	.06

For the first run (no covering), the sensitivity was 2.5686 mv/v, a little above the sensitivity requirement of 2 mv/v. The maximum non-linearity and hysteresis error was 0.11%. And, the zero shift was well below 1% of full scale.

After the .0625 inch covering was applied, the experimental gage was calibrated three times. A linear regression analysis was performed on each run with the results shown in Table 8. The data shows the .0625 inch cover degraded the gage performance significantly. The sensitivity was only 1.140 mv/v, far below the desired sensitivity. The zero shift and combined non-linearity and hysteresis were well above the desired values of 1%.

The .0625 inch covering was removed and replaced by a .0025 inch membrane. The pressure gage was recalibrated and a linear regression analysis was performed on the data. These results, also shown in Table 8, indicate the combined non-linearity and hystersis was 2.17%. A close examination of the actual calibration data points indicates that the hysteresis is low, the 2.17% is therefore due mostly to non-linearity. Also, the zero shift was very low and the sensitivity was 2.4379 mv/v. This sensitivity is 95% of the uncovered pressure gage sensitivity and is above our design goal of 2 mv/v.

The cantilever pressure gage performed much better with the .0025 inch cover than with the .0625 inch covering. However, the thin cover also restricted the motion of the diaphragm which caused some non-linearity (2.17% combined non-linearity and hysteresis error versus 0.11% for the uncovered gage). The linearity could be improved by reducing the sensitivity since the 2.4379 mv/v was higher than the required 2 mv/v for 1 psi.

SUMMARY AND CONCLUSIONS

In extending the design of the existing Bikini gage to the 1 psi range, three types of diaphragms were investigated: uniform thickness, tapered thickness, and the cantilever supported diaphragm.

Equations for the theoretical pressure response were developed for each type of diaphragm. The equations for the uniform thickness diaphragms showed that: (1) a very thin diaphragm is required for measuring 1 psi, and (2) the material which will result in a pressure gage with the best combination of linearity and sensitivity is one with a low modulus of elasticity.

Since plastics have lower elastic modulus than metals the use of plastics for diaphragm materials was investigated. Experimental gages were made from two plastics, acetal and polyvinyl chloride (PVC). The static performance of the plastic diaphragm gages was poor - the gages had large hysteresis errors and zero shifts, and poor repeatability. In addition, the sensitivities of the experimental gages were well below the sensitivities predicted by the theoretical equations. The low sensitivity is probably due to the diaphragm stiffening effects caused by the strain gage and the adhesive used to bond it to the thin diaphragm. The 5 psi acetal gages were dynamically calibrated in a shock tube.* They had very high natural frequencies (10 Khz or higher).

To examine the stiffening effect of the strain gage and adhesive, a "zero thickness" gage was made by cementing a diaphragm type foil strain gage (1/2 inch in diameter) across the open end of a hollow cylinder. In this gage, the strain gage itself is the diaphragm. Calibration of this pressure gage showed that if the sensitivity in its linear range (0 to 0.1 psi) were extrapolated to 1 psi, it would have a sensitivity of about 2.5 mv/v. Since its linearity fell off rapidly above 0.1 psi, and when cemented to an actual diaphragm would have a much lower sensitivity, this experiment indicates that a 1/2 inch diameter diaphragm type foil strain gage cannot be used to make a uniform thickness diaphragm 1 psi pressure gage with a sensitivity of 2 mv/v.

The equations for the tapered thickness diaphragm pressure gages showed that for the required 1 psi pressure range the diaphragm would be too thin for metals other than magnesium. If tapered thickness diaphragms were made from plastics the resulting gages would probably have the low sensitivities and other problems of the plastic uniform thickness diaphragm gages. Thus, no experimental gages were constructed with tapered diaphragms.

Design equations were developed for a cantilever supported diaphragm gage. And, an experimental gage was built using beryllium copper. Rubber membrane coatings of two different thickness were used with this gage. The best performance of all the experimental gages was obtained with the cantilever supported diaphragm covered with a .0025 inch waterproof rubber membrane.

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* The DTNSRDC shock tube cannot be used below 5 psi.

This gage had the highest sensitivity of any of the experimental units 2.44 mv/v, exceeding the desired sensitivity of 2 mv/v. The cantilever supported diaphragm pressure gage also had a very low zero shift. The combined linearity and hysteresis of this gage was 2.17% of full scale, higher than the design goal of 1%.

A summary of the static calibrations of all the gages is given in Table 9. This table also contains the resonant frequency of the 5 psi acetal gages.

RECOMMENDATIONS

The cantilever supported diaphragm pressure gage looks promising. Its drawbacks are its high non-linearity error and somewhat low calculated natural frequency. However, it is expected that further development work could minimize if not eliminate these deficiencies. In particular, it is recommended that the cantilever supported diaphragm gage's non-linearity be reduced by reducing the diaphragm's deflection by making the diaphragm thicker. Since this gage had a higher sensitivity than desired it is expected that the reduction in sensitivity caused by making the diaphragm thicker would not prevent the desired sensitivity of 2 mv/v from being achieved. The thicker diaphragm would also increase the gage's natural frequency. In addition, the non-linearity error could be reduced by optimizing the membrane used to cover and waterproof the diaphragm.

ACKNOWLEDGEMENTS

In performing this study and preparing this report I wish to acknowledge the following individuals and their contributions: Mr. Raymond J. Ratcliffe made most of the calculations and wrote the three appendices. Mr. Florian E. Frillman shared in the pressure gage design work and performed the calibrations. Mr. Mills Dean, III contributed many of the original concepts and was the main stimulus in launching this project. Mr. Robert T. Schwartz provided guidance and good advice for the duration of the work. He also edited the report and handled its publication after I left DTNSRDC. Mr. Paul Mazzi constructed the experimental pressure transducers. His exceptional skill was a real asset to this project.

TABLE 9 – COMPARISON OF THE PERFORMANCE OF EXPERIMENTAL 1 PSI PRESSURE GAGES

Pressure Gage	Measured Sensitivity for FS output mv/v	Measured Repeatability % FS	Max. Combined Non-Linearity & Hysteresis % FS	Max. Zero Shift % FS	Measured Resonant Frequency kHz
5 psi acetal [illegible]	.7525	4.7	2.41	3.7	12.5
5 psi acetal [illegible]	.8327	6.9	3.87	5.85	10.0
2 psi PVC	.7890	2.9	3.37	5.52	not available
1 psi PVC	.523	4.4	4.33	5.76	not available
0.8 psi Cant. (.0625" covering)	1.414	2.7	-3.50	1.00	not available
0.8 psi Cant. (.0025" membrane)	2.4379	not available	-2.17	0.06	not available
"Zero Thickness" 0.1 psi	.2537 (would extrapolate to 2.537 for 1 psi FS)	6.9	-1.37	0.86	not available

APPENDIX A

STRAIN DISTRIBUTION AND AVERAGE STRAIN IN A UNIFORM THICKNESS DIAPHRAGM

The uniform thickness diaphragm is shown in Figure 2 on page 6. The derivation of the design equations for the uniform thickness diaphragm are based on the following assumptions:

(1) diaphragm deflection is limited to the diaphragm thickness,

(2) the diaphragm is rigidly clamped along its entire periphery,

(3) diaphragm material exhibits perfect elastic behavior, and

(4) the strain gage and adhesive add negligible stiffening and mass to the diaphragm.

When a uniform thickness diaphragm is deflected in a direction normal to its surface, the strain is related to the radius of the curvature of the deflected diaphragm at the point of interest. Specifically, from reference 8, the expressions for radial and tangential strains are:

$$e_r = \frac{d}{r_r} \text{ and } e_t = \frac{d}{r_t} \qquad (A.1)$$

where r_r and r_t are radii of curvature in radial and tangential directions, respectively, and, d is the distance of the point in question from the neutral surface of the diaphragm. See Figure A.1.

For the case of strain at the surface of the diaphragm,

d = t/2. And, the strain equation (A.1) become

$$e_r = \frac{t}{2r_r} \text{ and } e_t = \frac{t}{2r_t} \qquad (A.2)$$

For symmetrical bending of circular plates due to uniform loading, the radii of curvature[9] are given by:

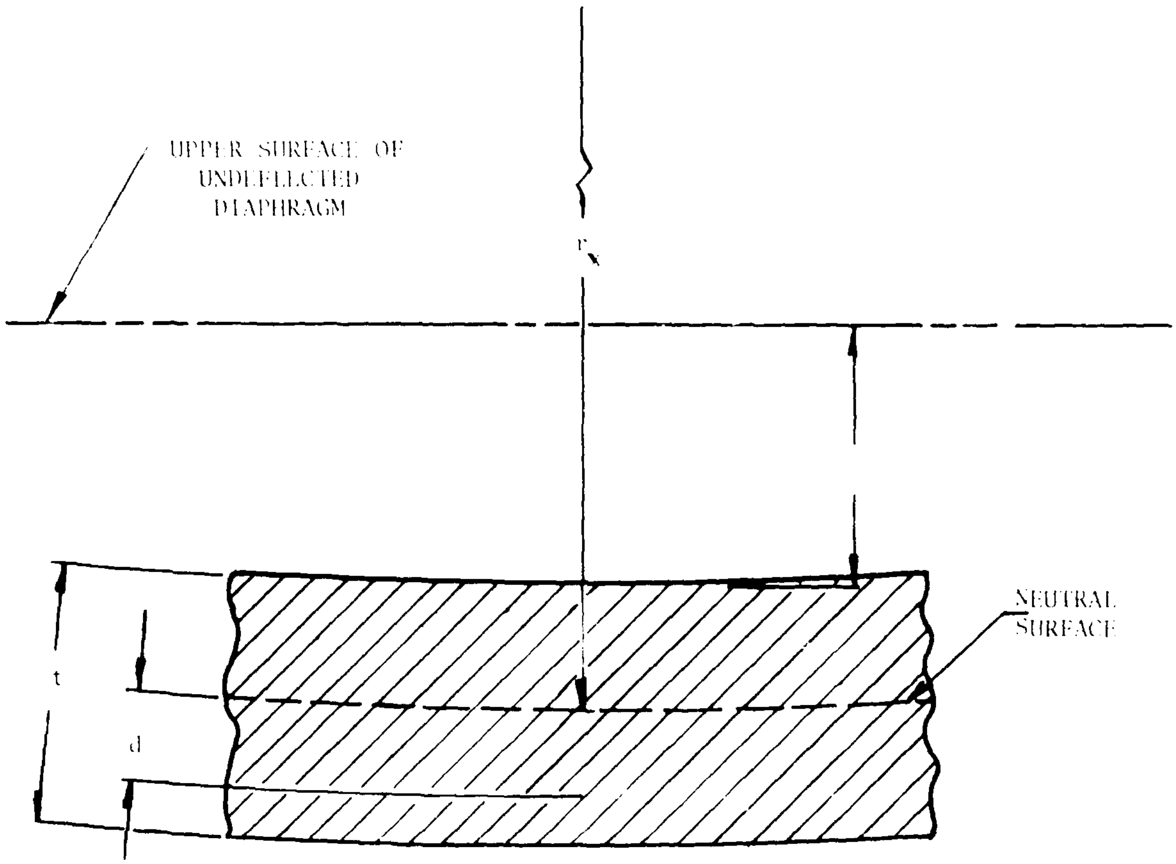


Figure A.1 - Cross-Section of Uniform Thickness Diaphragm

$$\frac{1}{r_r} = -\frac{d^2z}{dr^2}$$

and

$$\frac{1}{r_t} = -\frac{1}{r}\frac{dz}{dr} \tag{A.3}$$

where r = radial distance from the center of the diaphragm

z = the deflection of the diaphragm (see Figure A.1)

$\frac{dz}{dr}$ = the first derivative of deflection with respect to radial distance

$\frac{d^2z}{dr^2}$ = the second derivative

The deflection, z, of a circula plate with clamped edges which is uniformly loaded[10] is given by:

$$z = \frac{12P(1-v^2)}{64Et^3}(R^2 - r^2)^2 \tag{A.4}$$

where P = pressure

v = Poisson's ratio

E = modulus of elasticity

t = diaphragm thickness

R = diaphragm radius

r = radial distance from center of diaphgram

Taking the derivatives of the deflection, z, the expressions for the radii of curvature become

$$\frac{1}{r_r} = -\frac{3}{16}\frac{P(1-v^2)}{Et^3}(12r^2 - 4R^2)$$

and

$$\frac{1}{r_t} = -\frac{3}{16}\frac{P(1-v^2)}{Et^3}(4r^2 - 4R^2)$$

Substituting these equations in (A.2) gives:

$$\epsilon_r = -\frac{3}{32}\frac{P(1-\nu^2)}{Et^2}(12r^2 - 4R^2) \qquad \text{(A.5)}$$

and

$$\epsilon_t = -\frac{3}{32}\frac{P(1-\nu^2)}{Et^2}(4r^2 - 4R^2) \qquad \text{(A.6)}$$

These equations give the radial and tangential strain at any radius, r, on the diaphragm. These strain distributions are plotted in Figure A.2.

The average radial strain is found from Equation (A.5) by integrating between r_1 and r_2 (the radial limits of the radial element of the strain gage shown in Figure A.2). The average strain in the radial element is therefore

$$\epsilon_{a_r} = \frac{\int_{r_1}^{r_2} \epsilon_r \, dr}{r_2 - r_1}$$

$$\epsilon_{a_r} = \frac{1}{r_2 - r_1}\int_{r_1}^{r_2} -\frac{3}{32}\frac{P(1-\nu^2)}{Et^2}(12r^2 - 4R^2)\,dr$$

$$\epsilon_{a_r} = -\frac{3}{8}\frac{P(1-\nu^2)}{Et^2}\left[\frac{(r_2^3 - r_1^3) - R^2(r_2 - r_1)}{r_2 - r_1}\right] \qquad \text{(A.7)}$$

To put Equation (A.7) into a more useful form, an average value for the ratios $\frac{r_1}{R}$ and $\frac{r_2}{R}$ were computed based on the values of r_1 and r_2 of commercially obtainable diaphragm strain gage sizes.

The typical averages of r_1 and r_2 were found from manufacturer's data sheets to be

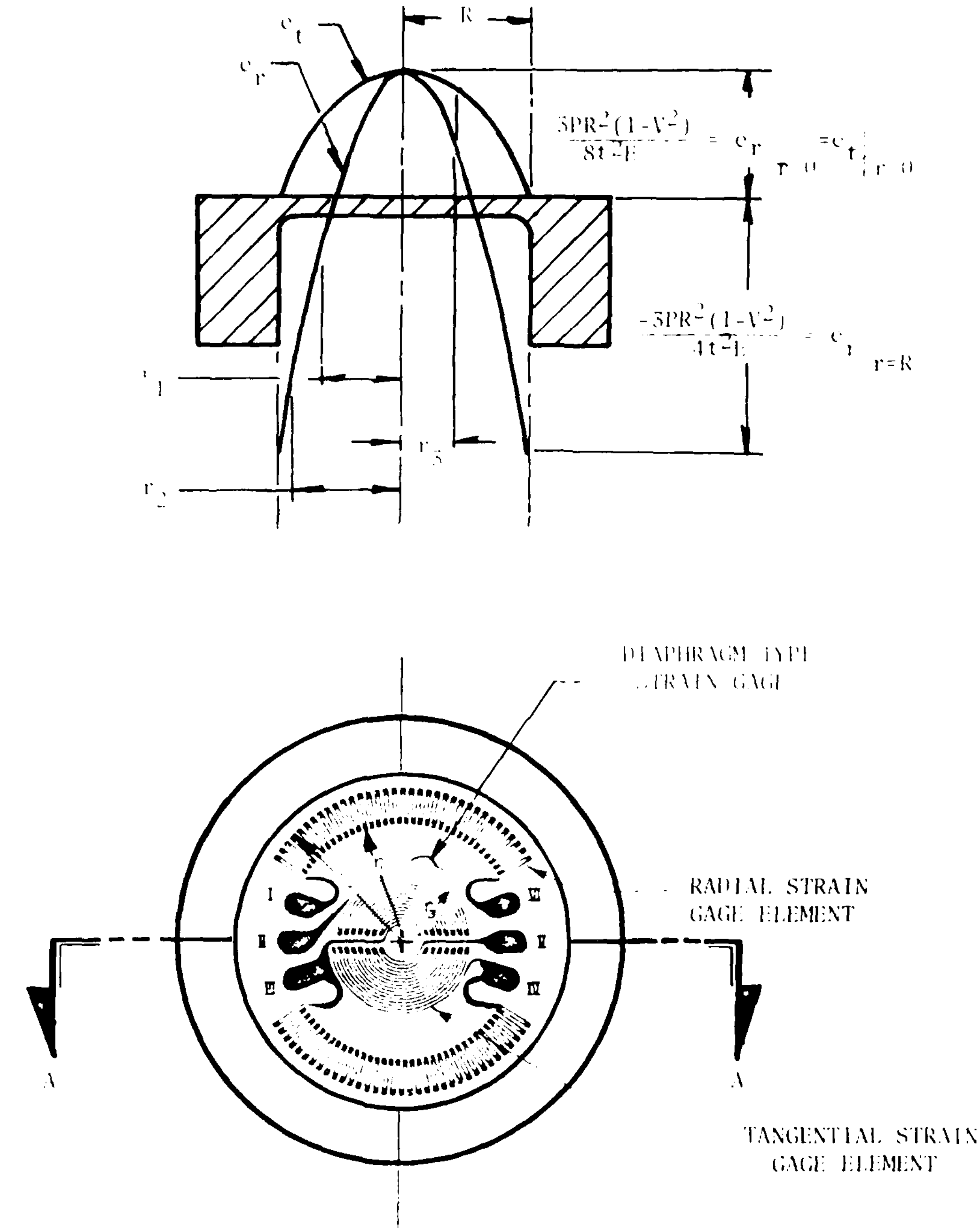


Figure V.2 - Strain Distribution in Uniform Thickness Diagphragm

$$r_{1_{AVG}} = .7852R$$

and

$$r_{2_{AVG}} = .9291R$$

Substitution of these values in Equation (A.7) gives a value of the average radial strain as

$$e_{a_r} = -.4534 \frac{PR^2(1-\nu^2)}{Et^2}$$

The average tangential strain is found from Equation (A.6)

$$e_{a_t} = \frac{\int_0^{r_3} e_t \, dr}{r_3}$$

where r_3 is defined in Figure A.2 as the maximum radial dimension of the tangential strain measuring element of the diaphragm foil strain gage.

$$e_{a_t} = \frac{1}{r_3} \int_0^{r_3} -\frac{3}{32} \frac{P(1-\nu^2)}{Et^2} (4r^2 - 4R^2) dr$$

$$e_{a_t} = -\frac{3}{32} \frac{P(1-\nu^2)}{Et^2} \left[\frac{4r_3^3 - 12R^2 r_3}{3r_3} \right] \qquad (A.8)$$

From manufacturer's data sheets it was found for the tangential elements that

$$r_3 = .4082R$$

Substitution of this value gives

$$e_{a_t} = .3542 \frac{PR^2(1-\nu^2)}{Et^2}$$

The average gage strain is

$$e_a = \frac{|e_{a_r}| + |e_{a_t}|}{2}$$

$$= \frac{.4534 + .3542}{2} \frac{PR^2(1-v^2)}{Et^2}$$

$$e_a = .4038 \frac{PR^2(1-v^2)}{Et^2} \qquad (A.9)$$

The output of a 4 active arm strain gage bridge[11] is given by:

$$\frac{V_o}{V} = 10^3 K e_a \text{ mv/v} \qquad (A.10)$$

when V_o = the bridge output voltage,

V = bridge excitation voltage, and

K = gage factor.

For $K=2$, and using Equation (A.9) for e_a gives

$$\frac{V_o}{V} = .81 \frac{PR^2(1-v^2)}{Et^2} \times 10^3 \text{ mv/v}$$

which agrees closely with Equation (4) of reference 12.

APPENDIX B

TAPERED THICKNESS DIAPHRAGM

Figure B.1 shows the cross section of a tapered thickness diaphragm. The variation in diaphragm thickness is given by the equation $t = t_o \exp(\frac{-nx^2}{6})$, see reference 13. Also shown in Figure B.1 are the diaphragm thicknesses and shapes for values of n ranging from +4 to -4. Note that for positive values of n the diaphragm is thickest at the center reducing in thickness to the edge. The condition of positive n would thus lead to a relatively low natural frequency since a large percentage of the diaphragm mass would be at the center supported by the relatively thin portion of the diaphragm at the ends. The opposite is true for conditions of negative n. The relatively small mass at the center and the thicker stiff ends would indicate a higher natural frequency than for a non-tapered diaphragm.

The following assumptions apply to the analysis of a variable thickness diaphragm:

1. the diaphragm is made of a homogeneous and isotropic material
2. the thickness is not greater than one-quarter (1/4) of the diameter, and the maximum deflection is not greater than the thickness, t_o, at the diaphragm center
3. all forces are normal to the plane of the diaphragm
4. the diaphram is nowhere stressed beyond its elastic limit

From reference 13, the maximum stress and deflection equations for a diaphram with shape $t = t_o \exp(\frac{-nx^2}{6})$

are

$$s = b\,\frac{PR^2}{t_o^2} \tag{B.1}$$

and

$$y = a\,\frac{PR^4}{Et_o^3} \tag{B.2}$$

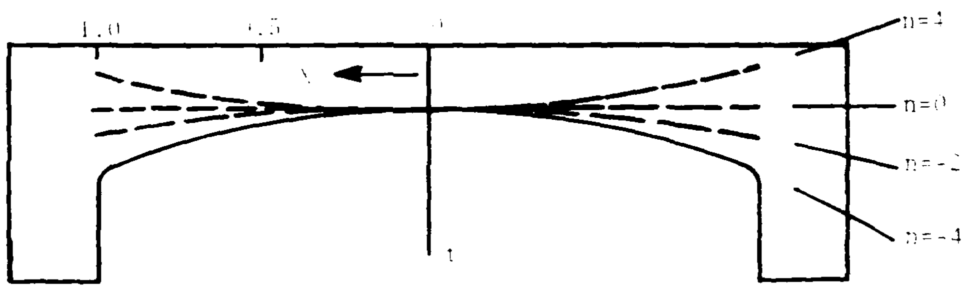


Figure B.1 - Cross-Section of Tapered Thickness Diaphragm

where z = deflection at the center of the diaphragm

R = diaphragm radius

t_o = thickness at the center of the diaphragm

P = pressure

E = modulus of elasticity

$x = r/R$

n = constant

a and b are tabulated constants in Table B.1

To limit the non-linearity error to 1% when the pressure is maximum ($P=P_{max}$), we have the restriction that $z_{max} = 1.0\ t_o$. Using Equation B.2 this gives

$$\frac{z_{max}}{t_o} = 1.0 = a\,P_{max}\,\frac{R^4}{Et_o^4} \qquad (B.3)$$

or

$$\left(\frac{R}{t_o}\right)^4 = \frac{E}{aP_{max}} \qquad (B.4)$$

Solving Equation (B.1) for the ratio $\left(\frac{R}{t_o}\right)^2$, squaring this result, and then using the relation $s = Ee$ (which ignores the small effect of Poisson's ratio) gives

$$\left(\frac{R}{t_o}\right)^4 = \left(\frac{s_{max}}{bP_{max}}\right)^2 = \left(\frac{Ee_{max}}{bP_{max}}\right)^2 = \frac{E}{aP_{max}}$$

(Note the last equality comes from Equation (B.4)). Using the last two terms gives:

$$E = \frac{b^2 P_{max}}{a e_{max}^2} \qquad (B.5)$$

Table B.1 shows values for E tabulated from Equation (B.5) for e_{max} = 500 microinch/inch, P_{max} = 1.0 psi, z_{max} = 1.0 t_o . The table is taken from page 228 of reference 13.

TABLE B.1 - MODULUS OF ELASTICITY REQUIREMENTS FOR A TAPERED THICKNESS DIAPHRAGM

n	+4	+3	+2	+1	0	-1	-2	-3	-4
b	2.14	1.63	1.31	0.985	0.75	0.55	0.43	0.32	0.26
a	0.4375	0.3490	0.276	0.217	0.1707	0.1343	0.1048	0.0830	0.0653
$F (\times 10^{6})$	41.9	30.5	24.9	17.9	13.2	9.0	7.0	4.9	4.1
R_o/t_o	98.9	96.7	97.4	95.3	93.7	90.5	90.4	87.8	45.1

Note: For magnesium E = 6.5 x 10^6 , this implies R/t_o = 90.4 or

t_o = (0.25/90.4) = .00276 inch.

The study of the minimum required elastic modulus for tapered diaphragms with a thickness variation of $t = t_o \exp \frac{nx^2}{6}$ indicates that for a diaphragm thinner at the center than at the edges ($n < 0$), an elastic modulus of less than 10 million is necessary. A suitable material would be a magnesium alloy where $F = 6.5 \times 10^6$ psi. Information on the corrosion of magnesium alloys however indicates this material is not suitable for aqueous environments.

APPENDIX C

CANTILEVER BEAM DIAPHRAGMS

These assumptions apply throughout the following analysis (see reference 14):

1. the beam is of a homogeneous material
2. the undeflected beam is straight
3. the beam cross section is uniform along its length
4. the beam has at least one longitudinal plane of symmetry
5. all loads and reactions are perpendicular to the beam axis and lie in the same plane
6. the length-to-thickness ratio of the beam is 8 or more
7. the beam is not disproportionately wide
8. the maximum stress does not exceed the proportional limit of the material

The diaphragm shown in Figure C.1 is modeled as a double cantilever beam uniformly loaded and with both ends fixed.

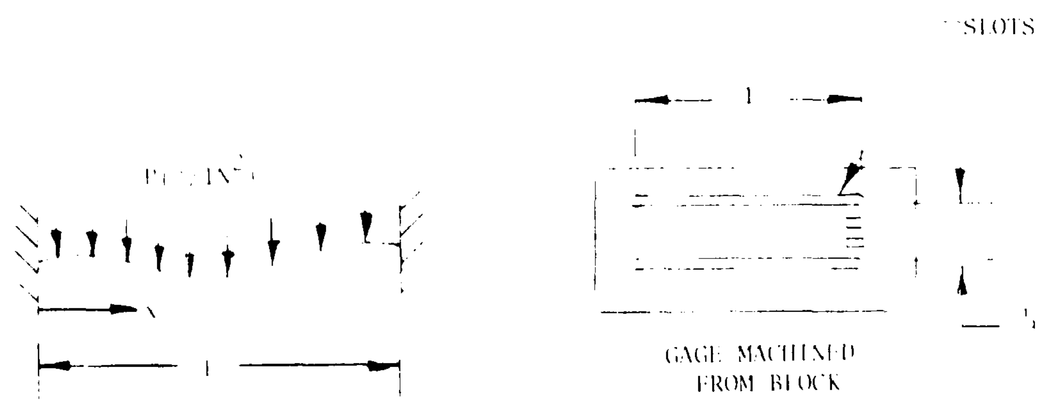


Figure C.1 - Double Cantilever Beam Diaphragm, Type One

The applicable bending moment equation[15] is

$$M = \frac{1}{2} PLw \left[x - \frac{x^2}{L} - \frac{1}{6} L \right] \tag{C.1}$$

where P = pressure on diaphragm

L = length of beam

w = width of beam

x = distance along beam

The stress in the outer fiber of the beam is

$$s = \frac{M}{I/C} = \frac{6M}{wt^2} \tag{C.2}$$

where I/C = section modulus

= $wt^2/6$ (see reference 16)

t = beam thickness

Since stress is proprotional to strain

$$e = \frac{s}{E} \tag{C.3}$$

Substituting Equation (C.2) into Equation (C.3) gives

$$e = \frac{6M}{wt^2 E} \tag{C.4}$$

Next, substituing Equation (C.1) into Equation (C.4) gives

$$e = \frac{3PL(x - \frac{x^2}{L} - \frac{1}{6}L)}{t^2E} \qquad (C.5)$$

where e = strain in the outer fibers of the beam

E = modulus of elasticity of the beam material

An equation for the thickness is found by rearranging Equation (C.5).

$$t = \left[\frac{3PL(x - \frac{x^2}{L} - \frac{L}{6})}{Ee} \right]^{1/2}$$

With (1) an assumed strain gage location of x = .030 inches, (2) an elastic modulus of $E = 19 \times 10^6$ psi for beryllium copper, (3) beam dimensions shown in Figure C.1, and (4) a strain of 1000 microinches/inch.

$$t = \left[\frac{3(1.0)(1.0)(.030 - \frac{(.030)^2}{1.0} - \frac{1.0}{6})}{1.0 \times 10^{-3} \times 19 \times 10^6} \right]^{1/2}$$

t = .0047 inches

A diaphragm with this thickness was too thin to machine and gage. Therefore, a second type of double cantilever beam diaphragm shown in Figure C.2 was investigated.

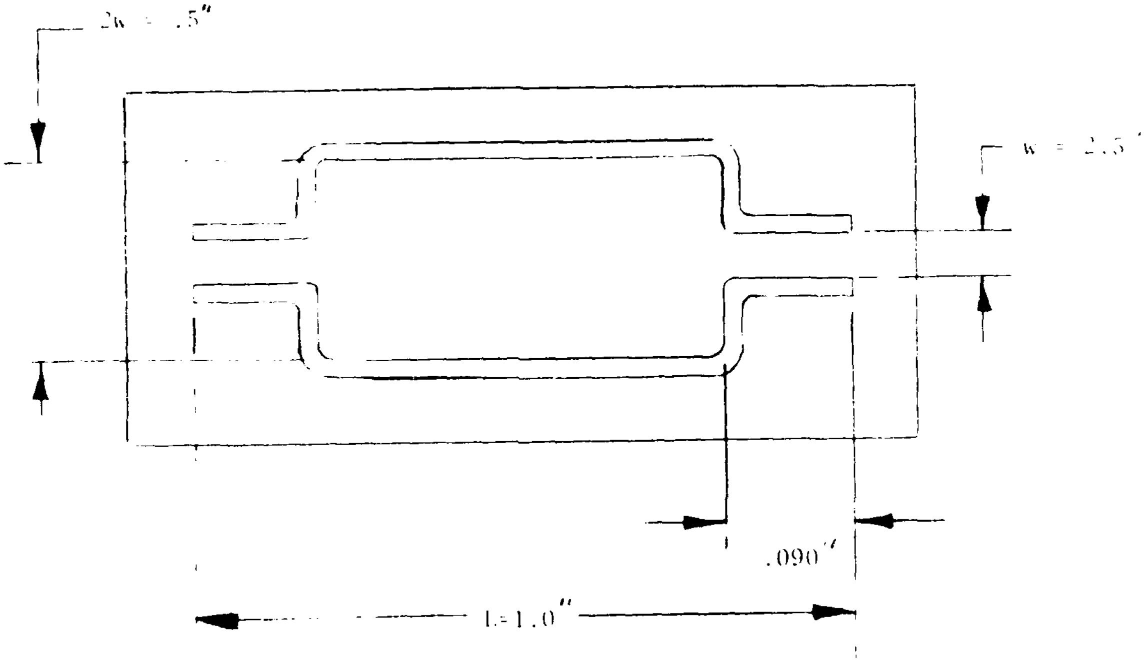
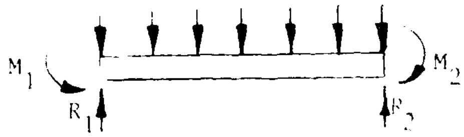


Figure C.2 - Double Cantilever Beam Diaphragm, Type Two

The analysis of moments and end reactions is performed by the method of superposition using cases 33 and 34 in reference 15:

case 33: for fixed ends and uniform loading (In the following derivation w = .25 inch L = 1.0 inch, and w and L are defined as in Figure C.2)

$$F = PwL$$



$$R_1 = R_2 = 1/2\ PwL$$

$$= .125P$$

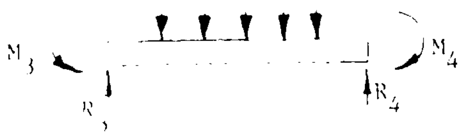
$$M_1 = -\frac{1}{12}\ PwL^2$$

$$M_1 = -.021P$$

$$M_2 = +.021P$$

case 34: for fixed ends and partial uniform loading

$$F = .82\ PwL$$



$$R_3 = R_4 = \frac{1}{4}\ \frac{.82Pw}{L}\ \left(12d^2 - 8\ \frac{d^3}{L} + 2\ \frac{bc^2}{L} - \frac{c^3}{L} - c^2\right)$$

where $b = .91$
$c = .82$
$d = .50$

$$R_3 = R_4 = .1025P$$

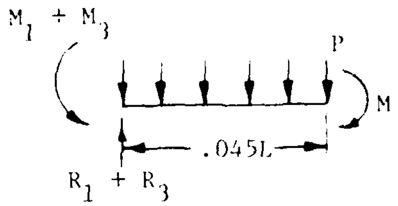
$$M_3 = -\frac{1}{24} \; .82Pw \left(24 \frac{d^3}{L} - 6 \frac{bc^2}{L} + \frac{3c^3}{L} + 4c^2 - 24d^2\right)$$

$$M_3 = -.0199P$$

$$M_4 = +.0199P$$

where M_3 and M_4 have the directions shown above.

Summing moments at a gage location of x = .045 inches as shown below



gives

$$M - M_1 - M_3 + .045L\,(R_1 + R_3)$$

$$-\frac{(.045)^2}{2} \; L^2Pw = 0$$

or

$$M = -.0509P$$

Substituting the above equation into Equation (C.4) and solving for t gives

$$t = \left[\frac{6(.0509P)}{wE\epsilon}\right]^{\frac{1}{2}}$$

for w = .25 inches

P = 1.0 psi

ϵ = 1.0×10^{-3}

E = 19×10^6

$$t = .0080 \text{ inch}$$

This diaphragm is likewise thinner than desired.

A third fixed end double cantilever beam diaphragm is shown in Figure C.3.

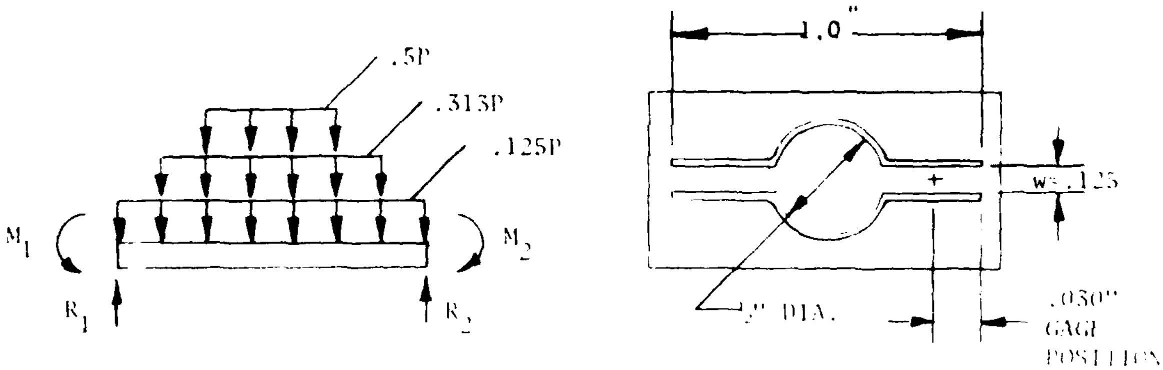
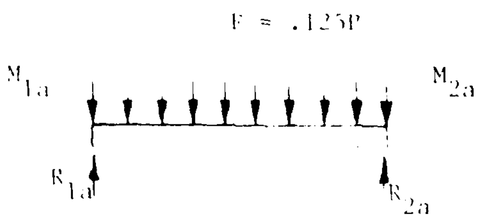


Figure C.3 - Double Cantilever Beam Diaphragm, Type Three

To simplify the calculations the stepped loading shown was used.

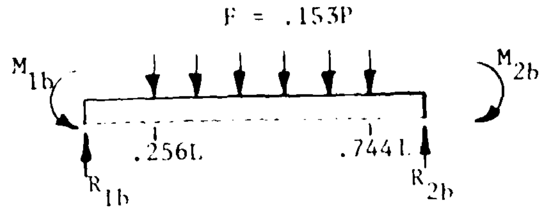
The moments and reactions R_1, R_2, M_1, and M_2 were obtained by the principle of superposition from the free body diagrams that follow:



from case 33 (see reference 15)

$$R_{1a} = \frac{1}{2} F = .0625P$$

$$M_{1a} = -\frac{1}{12} FL = -.0104P \qquad \text{(counter clockwise)}$$



from case 34 (see reference 15)

$$R_{1b} = \frac{1}{4}\,\frac{F}{L^2}\left(12d^2 - 8\,\frac{d^3}{L} + 2\,\frac{bc^2}{L} - \frac{c^3}{L} - c^2\right)$$

where $b = .744$

$c = .488$

$d = .500$

then

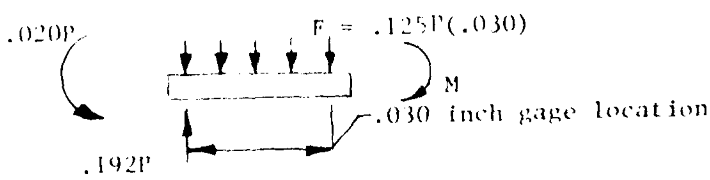
$$R_1 = R_{1a} + R_{1b} + R_{1c}$$

$$R_1 = .192P$$

and

$$M_1 = M_{1a} + M_{1b} + M_{1c}$$

$$= .020P$$



Summing moments at the gage location x = .030

$$M = .125P\,\frac{(.030)^2}{2} - .020P - .192P\,(.030)$$

$$M = -.0258P$$

indicating clockwise moment.

Substituting this expression for M into Equation (C.4) gives

$$t = \left[\frac{6(.0258P)}{weE}\right]^{1/2}$$

for $P = 1.0$

$w = .125$ in.

$e = 1.0 \times 10^{-3}$

$E = 19 \times 10^{6}$

$$t = .0081$$

Again the diaphragm for this gage is too thin.

The analysis of the single cantilev r beam diaphragms begins with the arrangement shown in Figure C.4.

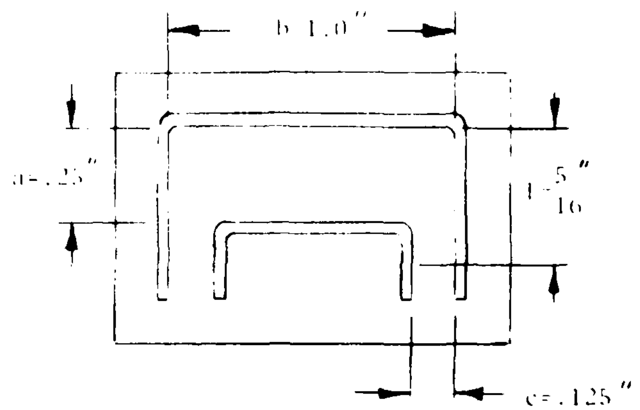


Figure C.4 - Single Cantilever Beam Diaphragm, Type One

The moment for a cantilever beam uniformly loaded is given in reference 15 case 3 as:

$$M = \frac{1}{2} FL$$

where F = abP = total load on the beam in pounds force

L = length of the loaded span

then

$$M = \frac{1}{2}(.25)(1.0)\ P(.3125)$$

$$M = .0391P$$

Making use of Equation (C.4)

where P = 1.0 psi

$w = 2c = .25$

$e = 1.0 \times 10^{-3}$

$E = 19 \times 10^{6}$

$$t = .0070 \text{ in.}$$

Again, this diaphragm is too thin.

A second single cantilever beam diaphragm is shown in Figure C.5.

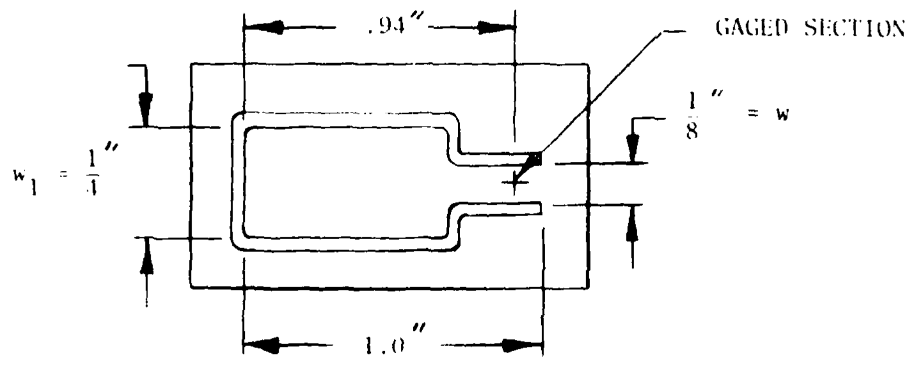


Figure C.5 - Single Cantilever Beam Diaphragm, Type Two

The assumption used to analyze this type of beam is that there is a uniform load along the beam from the free end to the gaged section.

Case 3 (see reference 15) for a cantilever beam with uniform loading gives

$$M = \frac{1}{2}\frac{F}{L}x^2$$

where F = total load on the beam, lbs

L = length of the beam

x = distance from free end of beam

With the assumption made above and using dimensions shown in Figure C.5 we have

$$F = wLP = .25P$$

and

$$M = \frac{1}{2}\frac{.25P}{1.0}(.94)^2 = .1105P$$

then from Equation (C.5)

$$t = \left[\frac{.6627P}{weE}\right]^{1/2}$$

substituting P = 1.0

w = .125

$e = 1.0 \times 10^{-3}$

$E = 19 \times 10^6$

gives

$$t = .0167 \text{ inches}$$

A diaphragm 0.0167 inches thick is reasonable for a pressure gage.

A resonant frequency calculation for a single cantilever beam diaphragm of this type was made. If it is assumed that the beam behaves like a uniform cantilever with a concentrated mass at the end, then

$$f_n = 0.28 \left[\frac{EI}{(M + \frac{33}{140} uL)\ L^3} \right]^{\frac{1}{2}} \text{ Hz} \tag{C.6}$$

where f_n = resonant frequency, Hz

E = modulus of elasticity

I = area moment of inertia of cantilever cross section

m = mass (pounds-sec^2/in) of the diaphragm

u = mass per unit length of cantilever

L = overall length of cantilever

Equation (C.6) is taken from reference 17.

The concentrated mass, m, is taken as the mass in excess of a 1/8 inch x 1.0 inch beam. The overall length of the beam is assumed to be the length from the geometric center of the ½ inch wide section to the fixed end.

then

$$I = 1/2\ wt^3 = .0485 \times 10^{-6}$$

where $m = 1.510 \times 10^{-6}$

$u = 1.605 \times 10^{-6}$

$L = .53$

substituting into (C.6) gives

$$f_n = 0.28 \left[\frac{19 \times 10^6 \times .0485 \times 10^{-6}}{(1.510 + \frac{33}{140} \times 1.6 \times 5.3)(.53)^3 \times 10^{-6}} \right]^{\frac{1}{2}}$$

$$f_n = 530 \text{ Hz}$$

This pressure gage would be reasonable to build.

The third single cantilever beam diaphragm considered (shown in Figure C.6) is simply a modification of the previous cantilever beam. Also shown in Figure C.6 is the total loading applied to the cantilever beam.

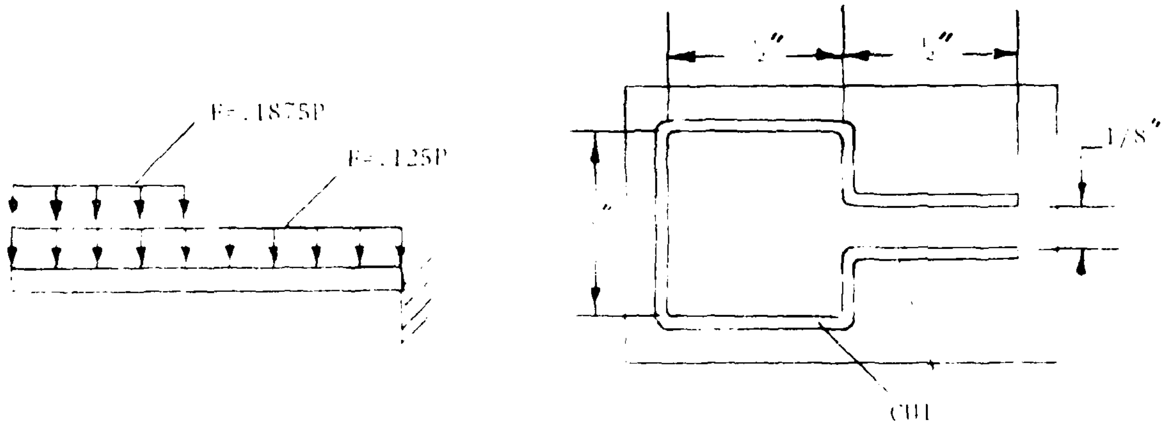
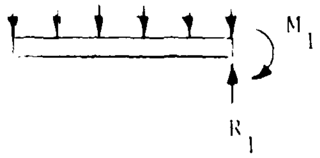


Figure C.6 - Single Cantilever Beam Diaphragm, Type Three

The gage location is assumed to be 0.97 inch from the free end and the moments can be computed applying the principle of superposition as before. Using case 3 from reference 15 gives a bending moment

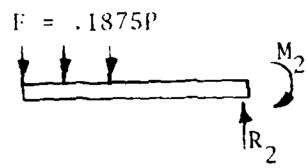
$$F = .125P$$



$$M_1 = \frac{1}{2} F_1 x^2$$

$$M_1 = .0588P$$

and Case no. 4 (see reference 15) gives the bending moment as



$$M_2 = \frac{1}{2} F \ (2x - 2L + a + b)$$

where $a = .50$

$b = 1.0$

$$M_2 = .1350P$$

The total moment at the gage section is

$$M = M_1 + M_2$$

$$M = .1938P$$

substituting $M = .1938$ (for $P = 1$ psi)

$w = .125$

$e = 1.0 \times 10^{-3}$

$E = 19 \times 10^6$

into Equation (C.4) gives

$$t = \left[\frac{6(.1938)}{.125 \times 1.0 \times 10^{-3} \times 19 \times 10^6} \right]^{\frac{1}{2}}$$

or

$$t = .0221 \text{ inch}$$

A resonant frequency calculation was also made for this diaphragm. Again assuming that the beam behaves like a uniform cantilever with a concentrated mass at the end, Equation (C.6) applies.

The values of the parameter subsituted in Equation (C.6) are

$m = 2.7 \times 10^{-6}$

$u = 1.8 \times 10^{-6}$

$I = 7.40 \times 10^{-8}$

$$f_n = 0.28 \left[\frac{19 \times 10^6 \times 11.11 \times 10^{-8}}{(3.09 + \frac{33}{140} \times 2.06 \times .75)(.75)^3 \times 10^{-6}} \right]^{\frac{1}{2}}$$

$$f_n = 340 \text{ Hz}$$

Even with the lower resonant frequency, this gage was the one chosen to be built due its greater thickness. Its performance is described in the section titled "CANTILEVER SUPPORTED DIAPHRAGM."

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